Geometric Sequences

Definitions and Re-Definitions

Geometric Sequence: A sequence that has the same ratio, **common ratio**, between any pair of consecutive terms.

Examples: 4, 8, 16, 32, 64 …

2000, 1000, 500, 250, 125 …

Recursive Sequence: A sequence for which one term (or more) is given and each successive term is determined from the previous term(s).

Examples: $4, 4 \times 2, (4 \times 2) \times 2, (4 \times 2 \times 2) \times 2 \ldots$

$a, a \times r, (a \times r) \times r, (a \times r \times r) \times r \ldots$

General Term: A formula, labelled t_n, that expresses each term of a sequence as a function of its position. For example, if the general term is $t_n = 2n$, then to calculate the 12^{th} term (t_{12}), substitute $n = 12$.

Examples: $t_n = 4 \times 2^{n-1}$

$t_n = a \times r^{n-1}$, a represents the first term, r represents the ratio of successive terms

Recursive Formula: A formula relating the general term of a sequence to the previous term(s).

Examples: $t_1 = 4$, $t_n = 4 \times t_{n-1}$, where $n > 1$

$t_1 = a$, $t_n = r \times t_{n-1}$, where $n > 1$

Worked Example

Determine the general term, the recursive formula, and the 10^{th} term in the sequence

3, -12, 48, -192, 768 …

The terms in the sequence are in a <u>ratio</u> of -4. Therefore this is a **geometric sequence**.

The formula for the general term of a geometric sequence is $\mathbf{t_n = a \times r^{n-1}}$.
For this example, a = 3 and r = -4.
Therefore, the general term for this example is $\mathbf{t_n = 3 \times (-4)^{n-1}}$.

The generalized recursive formula for a geometric sequence is $\mathbf{t_1 = a, t_n = rt_{n-1}}$**, where** $\mathbf{n > 1}$.
Therefore, the recursive formula for this example is $\mathbf{t_1 = 3, t_n = (-4)t_{n-1}}$**, where** $\mathbf{n > 1}$.

The 10^{th} term in this sequence is $\mathbf{t_n = 3 \times (-4)^{10-1} = 3 \times (-4)^9 = 3 \times -262{,}144 = \underline{-786{,}432}}$

Sequence	Ratio	General Term	Recursive Formula	10^{th} Term
5, 15, 45, 135 …				
10 125, 6 750, 4 500 …				
125, 50, 20, 8 …				
15, -60, 240 …				