What's Going On?

Checking In

Minds on — True or False?

Action! — Rationalizing

Consolidation — A Tricky One

Learning Goal - I will be able to rationalize a denominator by multiplying by the conjugate .

Minds on

True or False?

For each statement

i. determine if it is true or false.

ii. if it is false, give a counter example, if it is true provide 2 examples.

1. $\sqrt{x}+\sqrt{y}=\sqrt{x+y}$	2. $\sqrt{x}-\sqrt{y}=\sqrt{x-y}$
3. $\sqrt{x}\times\sqrt{y}=\sqrt{x\times y}$	4. $\sqrt{x}\div\sqrt{y}=\sqrt{x\div y}$
5. $\left(\sqrt{x}\right)^2=\sqrt{x^2}$	6. $a\sqrt{x}=\sqrt{a^2x}$

Minds on

True or False?

$$\sqrt{x} + \sqrt{y} = \sqrt{x+y}$$ False

$$\sqrt{x} - \sqrt{y} = \sqrt{x-y}$$ False

$$\sqrt{x} \times \sqrt{y} = \sqrt{x \times y}$$ True

$$\sqrt{x} \div \sqrt{y} = \sqrt{x \div y}$$ True

$$(\sqrt{x})^2 = \sqrt{x^2} = x$$

$$a\sqrt{x} = \sqrt{a^2 x} = a\sqrt{\cancel{a \times a} \times x}$$

Action!

Rationalizing

The process of changing a denominator from a radical to a rational number is called rationalizing the denominator. Later in the course, we will need to use these techniques to rationalize the numerator.

Action!

Rationalizing

One Term Radical in the Denominator

When the denominator is a one-term radical, we multiply both the numerator and the denominator by the denominator.

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ x } \overset{1}{\boxed{\frac{\sqrt{b}}{\sqrt{b}}}} = \frac{\sqrt{ab}}{b}$$

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

Action!

Rationalizing

One Term Radical in the Denominator

When the denominator is a one-term radical, we multiply both the numerator and the denominator by the denominator.

Example 0: Simplify $\frac{7}{\sqrt{5}}$ by rationalizing the denominator

$$\frac{7}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{5}$$

Action!

Rationalizing

One Term Radical in the Denominator

When the denominator is a one-term radical, we multiply both the numerator and the denominator by the denominator.

Example 1: Simplify $\frac{5}{8\sqrt{3}}$ by rationalizing the denominator

$$\frac{5}{8\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{24}$$

$$\frac{5}{8\sqrt{3}} \times \frac{8\sqrt{3}}{8\sqrt{3}} = \frac{40\sqrt{3}}{192}$$

$$= \frac{20\sqrt{3}}{96}$$

$$= \frac{10\sqrt{3}}{48}$$

$$= \frac{5\sqrt{3}}{24}$$

Action!

Rationalizing

Expand and Simplify

$(a + b)^2$ $= (a+b)(a+b)$

$= a^2 + 2ab + b^2$

$\boxed{(a + b)(a - b) = a^2 - b^2}$

Action!

Rationalizing

Two Term Radical in the Denominator

When the denominator is a two-term radical, we multiply the numerator and the denominator by the conjugate.

$$\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{1}{\sqrt{a}-\sqrt{b}} \times \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{a-b}$$

Action!

Rationalizing

Two Term Radical in the Denominator

When the denominator is a two-term radical, we multiply the numerator and the denominator by the conjugate.

Example 2: Simplify $\frac{6}{\sqrt{2}+\sqrt{3}}$ by rationalizing the denominator

$$\frac{6}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$$

$$=\sqrt{4}-\cancel{\sqrt{6}}+\cancel{\sqrt{6}}-\sqrt{9}$$

$$=2-3$$

$$=\frac{6(\sqrt{2}-\sqrt{3})}{2-3}$$

$$=\frac{6\sqrt{2}-6\sqrt{3}}{-1}$$

$$=\frac{6(\sqrt{2}-\sqrt{3})}{-1}$$

$$\boxed{=-6(\sqrt{2}-\sqrt{3})}$$

or

$$-6\sqrt{2}+6\sqrt{3}$$

$$\frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$$

$$= \frac{\cancel{3}(\sqrt{5}-\sqrt{2})}{\cancel{3}}$$

$$= \sqrt{5}-\sqrt{2}$$

Action!

Rationalizing

Two Term Radical in the Denominator

When the denominator is a two-term radical, we multiply the numerator and the denominator by the conjugate.

Example 3: Simplify $\frac{4}{3\sqrt{5}-1}$ by rationalizing the denominator

$$\frac{4}{3\sqrt{5}-1} \times \frac{3\sqrt{5}+1}{3\sqrt{5}+1}$$

$$= \frac{4(3\sqrt{5}+1)}{9 \times 5 - 1}$$

$$= \frac{4(3\sqrt{5}+1)}{44}$$

$$= \frac{3\sqrt{5}+1}{11}$$

$$\frac{4}{3\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$= \frac{4(\sqrt{5}+1)}{15+\underbrace{3\sqrt{5}-\sqrt{5}}_{2\sqrt{5}}-1}$$

$$=$$

Consolidation

A Tricky One

- perfect squares

Example 4: Simplify $\frac{3\sqrt{2}+2\sqrt{3}}{\sqrt{6}-\sqrt{3}}$ by rationalizing the denominator

$$\frac{3\sqrt{2}+2\sqrt{3}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} = \frac{3\sqrt{12}+3\sqrt{6}+2\sqrt{18}+2\sqrt{9}}{3}$$

$$= \frac{3\sqrt{12}+3\sqrt{6}+2\sqrt{18}+6}{3}$$

$$= \frac{3\sqrt{4\times 3}+3\sqrt{6}+2\sqrt{9\times 2}+6}{3}$$

$$= \frac{3\times 2\sqrt{3}+3\sqrt{6}+2\times 3\sqrt{2}+6}{3}$$

$$= \frac{6\sqrt{3}+3\sqrt{6}+6\sqrt{2}+6}{3}$$

$$\boxed{= 2\sqrt{3}+\sqrt{6}+2\sqrt{2}+2}$$

$$\frac{2\sqrt{2}+3\sqrt{3}}{\sqrt{8}+\sqrt{6}} \times \frac{\sqrt{8}-\sqrt{6}}{\sqrt{8}-\sqrt{6}}$$

$$= \frac{2\sqrt{16}-2\sqrt{12}+3\sqrt{24}-3\sqrt{18}}{8-6}$$

*all can be reduced

$$= \frac{8-2\sqrt{4\times 3}+3\sqrt{6\times 4}-3\sqrt{9\times 2}}{2}$$

$$= \frac{8-4\sqrt{3}+6\sqrt{6}-9\sqrt{2}}{2}$$

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a few from each