Learning Goal: I will be able to solve problems involving instantaneous and average rate of change using the difference quotient.

Minds On: Warm-up

Action: Note and examples

Consolidation: Another way

Minds On

Question from Yesterday

Using the definition of the slope of a tangent, determine the slope of the tangent to the curve **y = 2x² + 3x - 5** at the point where **x = 3**.

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

same as last semester

$$= \lim_{h \to 0} \frac{\left(2(3+h)^2 + 3(3+h) - 5\right) - \left(2(3)^2 + 3(3) - 5\right)}{h}$$

$$= \lim_{h \to 0} \frac{\left(2(9+6h+h^2) + 9 + 3h - 5\right) - (18+9-5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{18} + 12h + 2h^2 \cancel{+9} + 3h \cancel{-5} \cancel{-18} \cancel{-9} \cancel{+5}}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 + 15h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2h+15)}{\cancel{h}}$$

$$= 15$$

Minds On

Warm-up: A pebble is dropped from a cliff, 120 m high. After t seconds, the pebble is s metres above the ground, where $s(t) = 120 - 6t^2, 0 \le t \le 4$.

a) Calculate the average velocity of the pebble between the times t = 2 s and t = 4 s.

$$\frac{s(4)-s(2)}{4-2} = \frac{24-96}{2} = -36\text{ m/s}$$

b) Calculate the average velocity of the pebble between the times t = 2 s and t = 2.5 s.

$$\frac{s(2.5)-s(2)}{2.5-2} = \frac{82.5-96}{0.5} = -27\text{ m/s}$$

c) Explain why your answers for parts a) and b) are different.

a is over 2s
b is only over 0.5s

the object is accelerating over time

d) What would happen to the average velocity if we continued to shorten the time interval?

the average velocity would approximate/approach instantaneous velocity

e) What is the average velocity between the times t = 2 and t = 2 + h

$$\text{average velocity} = \frac{s(2+h)-s(2)}{(2+h)-2}$$

$$= \frac{120-6(2+h)^2-96}{2+h-2}$$

$$= \frac{120-6(4+4h+h^2)-96}{h}$$

$$= \frac{\cancel{120}\cancel{-24}-24h-6h^2\cancel{-96}}{h}$$

$$= \frac{-6h^2-24h}{h}$$

$$= \frac{-6\cancel{h}(h+4)}{\cancel{h}}$$

$$= -6(h+4)$$

$$= -6h-24$$

Action

Average Rate of Change:

The difference quotient $\frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{h}$ is called the average rate of change in y with respect to x over the interval from x = a to x = a + h.

Instantaneous Rate of Change:

$\lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$, provided that the limit exists.

Average Velocity:

The difference quotient $\frac{\Delta s}{\Delta t} = \frac{s(a+h) - \overset{s}{\cancel{f}}(a)}{h}$ is the average rate of change in displacement with respect to time over the interval from t = a to t = a + h.

Instantaneous Velocity:

$\lim\limits_{h \to 0} \frac{s(a+h) - \overset{s}{\cancel{f}}(a)}{h}$, provided that the limit exists.

Action

Example 1: A toy rocket is launched straight up so that its height s, in metre, at time t, in seconds, is given by $s(t) = -4t^2 + 14t + 5$. What is the velocity of the rocket at t = 4?

$$v(t) = \lim_{h \to 0} \frac{s(4+h) - s(4)}{(4+h) - 4}$$

$$s(4+h) = -4(4+h)^2 + 14(4+h) + 5$$
$$= -4(16 + 8h + h^2) + 56 + 14h + 5$$
$$= -64 - 32h - 4h^2 + 56 + 14h + 5$$
$$= -4h^2 - 18h - 3$$

$$s(4) = -4(4)^2 + 14(4) + 5$$
$$= -64 + 56 + 5$$
$$= -3$$

$$v(t) = \lim_{h \to 0} \frac{(-4h^2 - 18h - 3) - (-3)}{h}$$

$$= \lim_{h \to 0} \frac{-4h^2 - 18h}{h}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}(2\cancel{h} + 9)}{\cancel{h}}$$

$$\frac{h(-4h - 18)}{h}$$

$$= -18 \text{ m/s}$$

Action

Example 2: The total cost, in dollars, of manufacturing x calculators is given by $C(x) = 10\sqrt{x} + 1000$.

a) What is the total cost of manufacturing 100 calculators?

$C(100) = 1100$

b) What is the rate of change in the total cost with respect to the number of calculators, x, being produced when x = 100?

$$\lim_{h \to 0} \frac{C(100+h) - C(100)}{h}$$

$$= \lim_{h \to 0} \frac{10\sqrt{100+h} + 1000 - 1100}{h}$$

$$= \lim_{h \to 0} \frac{10\sqrt{100+h} - 100}{h} \times \frac{10\sqrt{100+h} + 100}{10\sqrt{100+h} + 100}$$

$$= \lim_{h \to 0} \frac{100(100+h) - 10000}{h(10\sqrt{100+h} + 100)}$$

$$= \lim_{h \to 0} \frac{100\cancel{h}}{\cancel{h}(10\sqrt{100+h} + 100)}$$

$$= \lim_{h \to 0} \frac{100}{10\sqrt{100+h} + 100}$$

$$= \frac{100}{10\sqrt{100} + 100}$$

$$= \frac{100}{200}$$

$$= 0.5$$

the cost is ~~changing~~ increasing at a rate of \$0.50 per calculater.

Consolidation

Another Way

Example from yesterday

Example 2: a) using the definition of the slope of a tangent, determine the slope of the tangent to the curve y = $-x^2 + 4x + 1$ at the point determined by x = 3.

In general, the velocity of an object at time $t = a$ is $v(a) = \lim_{t \to a} \frac{s(t) - s(a)}{t - a}$. Similarly, the instantaneous rate of change in $y = f(x)$ with respect to x when $x = a$ is $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

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1, 7, 8, 10, 11, 13, 20