Learning Goal: I will be able to simplify and evaluate limits using the limit properties.

Minds On: What do you notice

Action: Note and examples

Consolidation: Exit Question

Minds On

Sketch It!

On your way in, grab a whiteboard, marker and eraser.

Sketch a function with each limit as given below.

$$\lim_{x \to 0} f(x) = 3$$

Minds On

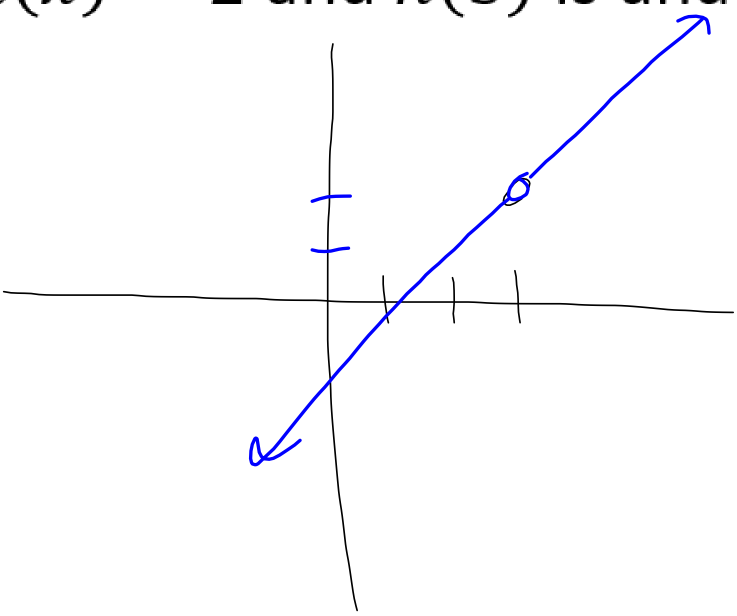
Sketch It!

$$\lim_{x \to 1} g(x) = DNE$$

Minds On

Sketch It!

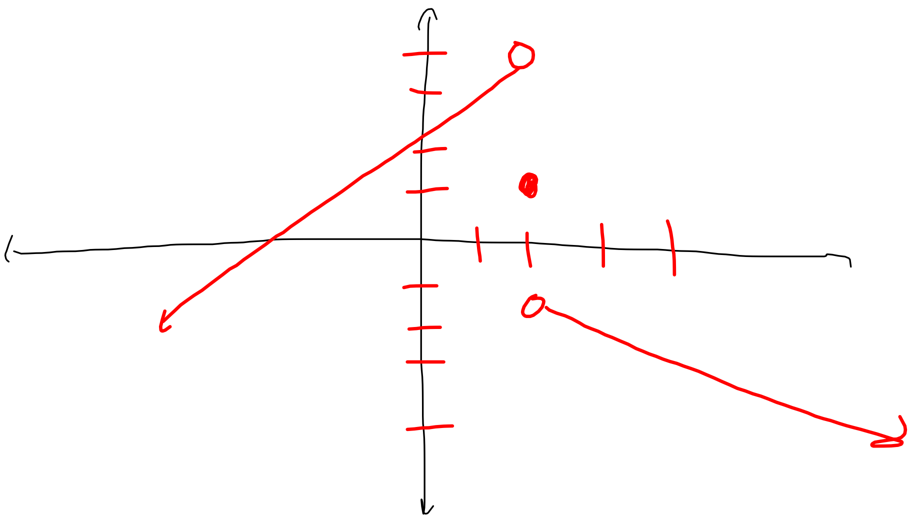
$\lim_{x \to 3} h(x) = 2$ and $h(3)$ is undefined



Minds On

Sketch It!

$$\lim_{x \to 2^-} m(x) = 4, \lim_{x \to 2^+} m(x) = -2 \text{ and } m(2) = 1$$



Action

What's the Relationship?

$4 \cdot \lim_{x \to 2} (x)$

Determine each limit.

$$\lim_{x \to 2} 3x^2$$

$= 12$

$$\lim_{x \to 2} 4x$$

$= 8$

$$\lim_{x \to 2} -1$$

$= -1$

$$\lim_{x \to 2} (3x^2 + 4x - 1)$$

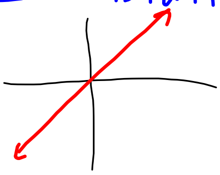
$= 19$

Action

Properties of Limits

1. $\lim_{x \to a} k = k$ ← constant

2. $\lim_{x \to a} x = a$



3. $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

4. $\lim_{x \to a}[cf(x)] = c[\lim_{x \to a} f(x)]$, for any constant c

5. $\lim_{x \to a}[f(x)g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)]$

6. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided that $\lim_{x \to a} g(x)$ does not equal 0

* 7. $\lim_{x \to a}[f(x)]^n = [\lim_{x \to a} f(x)]^n$, for any rational number n

$n = 2, 3, \ldots \quad n = \frac{1}{2}$

If f is a polynomial function, then $\lim_{x \to a} f(x) = f(a)$

Action

Example 2: Evaluate $\lim\limits_{x\to 3}(4x^3 - 2x - 5)$ using the properties of limits

$$= \lim_{x\to 3}(4x^3) + \lim_{x\to 3}(-2x) + \lim_{x\to 3}(-5)$$

$$= 108 + (-6) + (-5)$$

$$= 97$$

Action

Example 3: Evaluate $\lim_{x\to -1} \frac{x^2 - 5x+2}{2x^3 + 3x+1}$ using the properties of limits

$$= \frac{\lim_{x\to -1} (x^2-5x+2)}{\lim_{x\to -1} (2x^3+3x+1)}$$

$$= \frac{8}{-4}$$

$$= -2$$

Action

Example 4: Evaluate $\lim_{x\to 5}\sqrt{\frac{x^2}{x-1}}$ using the properties of limits

$$= \sqrt{\frac{\lim_{x\to 5} x^2}{\lim_{x\to 5} x-1}}$$

$$= \sqrt{\frac{25}{4}}$$

$$= \frac{5}{2}$$

Action

Sometimes we can't evaluate the limit using direct substitution, because it will result in an ***indeterminate form (0/0).*** In these cases, we find an equivalent fraction that works for all values of f except at x = a.

Example 5: Evaluate $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}}$$

$$= 4$$

Action

Example 6: Evaluate $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} \times \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$

$$= \lim_{x \to 0} \frac{\cancel{x} \cancel{+1} \cancel{-1}}{\cancel{x}(\sqrt{x+1}+1)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+1}+1}$$

$$= \frac{1}{2}$$

Action

Example 7: Evaluate $\lim\limits_{x \to 0} \frac{(x+8)^{\frac{1}{3}}-2}{x}$ Use substitution to simplify.

Use a substitution strategy.

Let $u=(x+8)^{1/3}$ as $x \to 0$
$u \to 2$

$u^3 = x+8$

so $x = u^3 - 8$

$$= \lim_{u \to 2} \frac{u-2}{u^3-8}$$

difference of cubes

$$= \lim_{u \to 2} \frac{\cancel{u-2}}{\cancel{(u-2)}(u^2+2u+4)}$$

$$= \lim_{u \to 2} \frac{1}{u^2+2u+4}$$

$$= \frac{1}{12}$$

Use a substitution strategy when you can't rationalize or factor.

Action

Factoring a Sum or Difference of Cubes

Factoring a Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Factoring a Difference of Cubes:

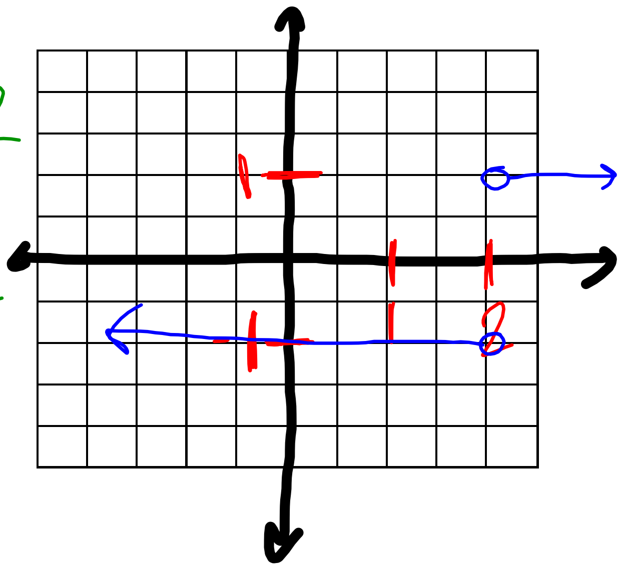
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Action

Example 8: Evaluate $\lim\limits_{x \to 2} \frac{|x-2|}{x-2}$. Illustrate with a graph.

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{(x-2)}{x-2} & \text{when } x>2 \\ \frac{-(x-2)}{(x-2)} & \text{when } x<2 \end{cases}$$

$$= \begin{cases} 1 & \text{when } x>2 \\ -1 & \text{when } x<2 \end{cases}$$



$$\lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1 \qquad \lim_{x \to 2^+} \frac{|x-2|}{x-2} = 1$$

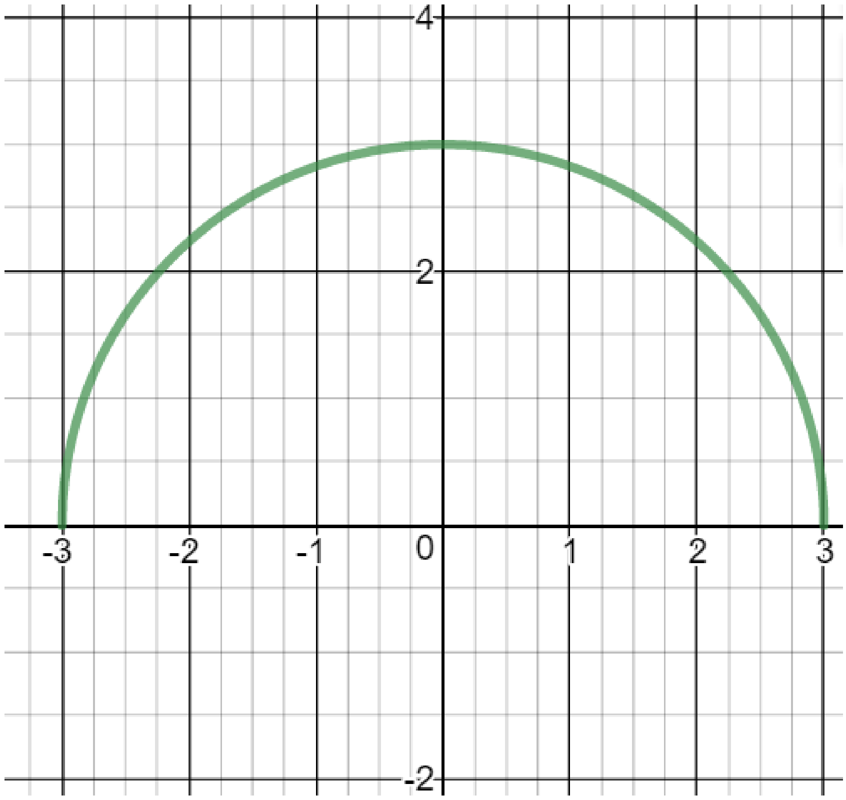
$$\therefore \lim_{x \to 2} \frac{|x-2|}{x-2} = \text{DNE}$$

Action

Example 9: a) Evaluate $\lim_{x \to 3-} \sqrt{9 - x^2}$. You may use a graphing technique.

b) Explain why the limit as x approaches 3+ cannot be determined.

c) What can you conclude about $\lim_{x \to 3} \sqrt{9 - x^2}$?



a) $\lim_{x \to 3^-} \sqrt{9-x^2} = 0$

b) the function does not exist when $x > 3$

c) the limit does not exist!

Consolidation

Practice

Pg. 45

3, 4, 7, **8**, 9