**Learning Goal:** I will be able to determine the derivative of a polynomial function by first principles.

**Minds On:** Slope of the tangent

**Action:** Note and examples

**Consolidation:** Exit Question

## Minds On

# Pushing RAFT

In this course, we will push RAFT until the end of the period.

This will ensure we get through the content for the day and will allow you to have time to practice or ask for help if you so choose.

***Tomorrow you will be in room 126 for the bulk of RAFT at the beginning of the period.**

## Minds On

# A Few Linear Things

Parallel Slopes

$y = 3x + 1$

slope = 3 parallel slope = 3

Perpendicular Slopes

$y = 3x + 1$ perpendicular slope = $-\frac{1}{3}$

Slope y-Intercept Form

$y = mx + b$

Point Slope Form

$y - y_1 = m(x - x_1)$ $y - 4 = 3(x - 1)$

Standard Form

$Ax + By + C = 0$

$3x + 2y + 4 = 0$

## Minds On

# Slope of the Tangent

$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Determine the slope of the tangent to the curve $f(x) = x^2$, at the point where x =

| | | |
|---|---|---|
| -2 | 0 | 1 |
| $m = -4$ | $m = 0$ | $m = 2$ |

## Minds On

# 2.1 The Derivative Function

When we were determining slopes of tangents (instantaneous rate of change) in the last unit, we looked at the average rate of change between two points, as the distance, h, between these points approached zero. This limit is a central part of calculus, so let's define it:

The **derivative** of f at the number a is given by $\boxed{f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}$, provided that this limit exists. The notation f'(a) is used to show the derivative of f(x) at x = a.
We pronounce it "f prime of a".

# Action

**Example 1:**

a) Determine the derivative of f(x) = $x^2$ at an arbitrary value of x.

b) Determine the slopes of the tangents to the parabola y = $x^2$ at x = -2, 0 and 1.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - (x)^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2x\cancel{h} + h^{\cancel{2}} \cancel{- x^2}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

b)

$$f'(-2) = 2(-2)$$
$$= -4$$

$$f'(0) = 2(0)$$
$$= 0$$

$$f'(1) = 2(1)$$
$$= 2$$

If we use an arbitrary "a" value to get the derivative, we'll get an answer that's a function rather than a numerical answer. This is called the **derivative function**:

| |
|---|
| The derivative of f(x) with respect to x is the function f'(x), where $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, provided that this limit exists. |

***Important Note (don't forget for your exam): When we use this limit to determine the derivative of a function, it is called determining the derivative from first principles.**

**Example 2:** Determine an equation of the tangent to the graph of $f(x) = \frac{1}{x}$ at the point x = 2.

Find slope by finding $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{x} - (\cancel{x}+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h)} \times \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x^2}$$

$y = mx + b$

when $x = 2$

$m = -\frac{1}{4}$

when $x = 2$

$y = \frac{1}{2}$

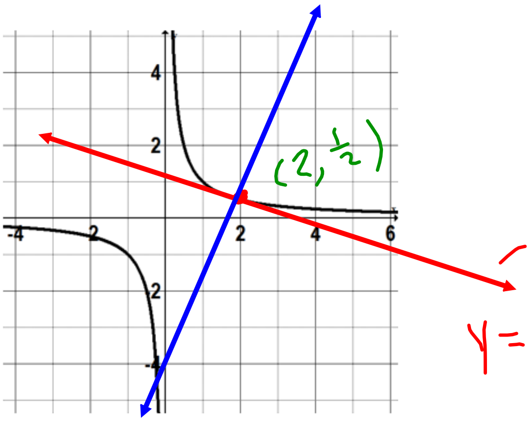
$b = y - mx$

$b = \frac{1}{2} - \left(-\frac{1}{4}\right)(2)$

$b = 1$

$y = -\frac{1}{4}x + 1$

**Example 3:** Determine an equation of the line that is perpendicular to the tangent to the graph of $f(x) = \frac{1}{x}$ at x = 2 and intersects it at the point of tangency.




tangent line

$y = -\frac{1}{4}x + 1$

$y = 4x + b$

$b = y - mx$

$b = \frac{1}{2} - (4)(2)$

$b = \frac{1}{2} - 8$

$b = \frac{1}{2} - \frac{16}{2}$

$b = \frac{-15}{2}$

$$\boxed{y = 4x - \frac{15}{2}}$$

or

$y = 4x - 7.5$

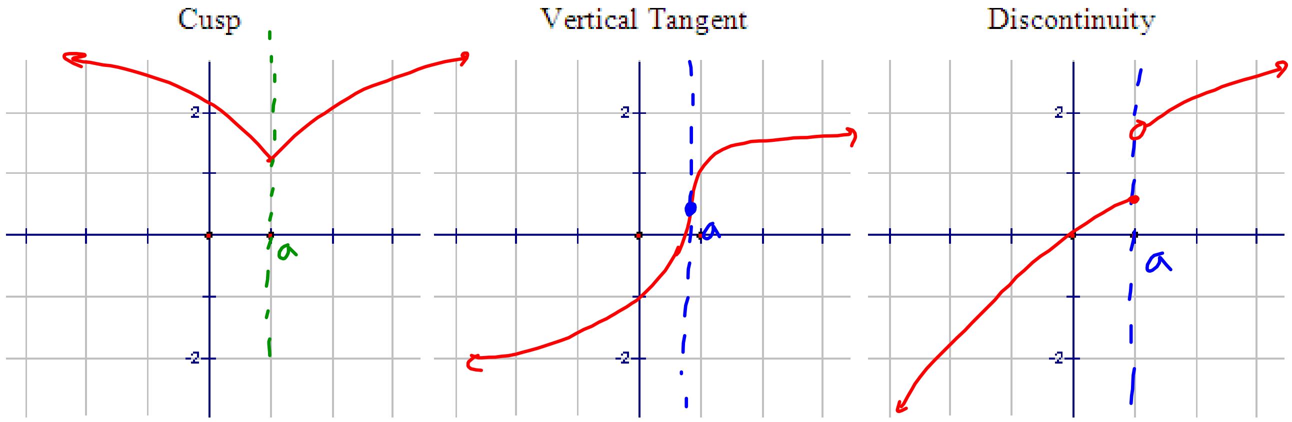
The line we found in example 3 has a proper name:

The **normal** to the graph of f at point P is the line that is perpendicular to the tangent at P.

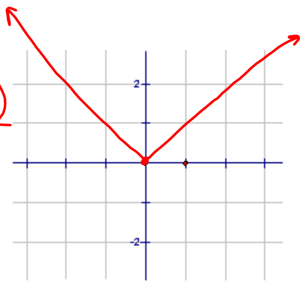
*The Existence of Derivatives*

We say that a function f is differentiable at "a" if f'(a) exists. At points where f is ~~not differentiable,~~ we say that the derivative does not exist. There are 3 common ways for a derivative to fail to exist:

can't determine the slope



**Example 4:** Show that the absolute value function $f(x) = |x|$ is not differentiable at x = 0.




$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \to 0} \frac{|h|}{h}$$

when $h < 0$, $|h| = -h$

when $h > 0$, $|h| = h$

so $\lim_{h \to 0^-} \frac{|h|}{h}$

$= \lim_{h \to 0^-} \frac{-h}{h}$

$= -1$

$\lim_{h \to 0^+} \frac{|h|}{h}$

$= \lim_{h \to 0^+} \frac{h}{h}$

$= 1$

limit doesn't exist

$\therefore f'(0)$ does not exist

*Other Notation for Derivatives*

The most commonly used notation for derivatives if f'(x) or y'. We can also use the symbol $\boxed{\frac{dy}{dx}}$ instead of f'(x).

The $\frac{dy}{dx}$ is called Leibniz notation, and is ***not a fraction***.

$\frac{\text{very small change in } y}{\text{very small change in } x}$

# Consolidation

**<u>Exit Question:</u>** Determine the derivative f'(t) of the function $f(t) = \sqrt{t}$, $t \geq 0$.

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{t+h} - \sqrt{t}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{t+h} - \sqrt{t}}{h} \times \frac{\sqrt{t+h} + \sqrt{t}}{\sqrt{t+h} + \sqrt{t}}$$

$$= \lim_{h \to 0} \frac{(\cancel{t} + \cancel{h}) - \cancel{(t)}}{\cancel{h}(\sqrt{t+h} + \sqrt{t})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{t+h} + \sqrt{t}}$$

$$= \frac{1}{2\sqrt{t}}, \; t > 0$$

## Consolidation

# Practice

# Pg. 73

# 5 - 7, 10, 15, 19