Learning Goal: I will be able to use the product rule to determine derivatives.

Minds On: What do you notice

Action: Note and examples

Consolidation: Exit Question

Minds On

Textbook Questions

From now on, I won't be assigning specific textbook problems for practice.

Basically, you should be able to do all of the Part A and Part B questions in each section as well as most of the Part C questions.

You should focus in on the areas that **you** need to practice. (The Basics, Word Problems, "Thinking" Questions)

Look through the questions and make sure you know how you would go about solving each one. If you find yourself unsure, try it!

Let's look at the questions in section 2.2.

Minds On

Whiteboard Derivatives

$f(x) = 4x^3$

$f'(x) = 12x^2$

Minds On

Whiteboard Derivatives

$$f(x) = -2x^2 + 4x^1 + 3$$

$$f'(x) = -4x + 4$$

Minds On

Whiteboard Derivatives

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \boxed{\frac{1}{2}x^{-\frac{1}{2}}} = 0.5x^{-0.5}$$

$$= \frac{1}{2x^{1/2}} = \boxed{\frac{1}{2\sqrt{x}}}$$

Minds On

Whiteboard Derivatives

$$f(x) = \sqrt[3]{x} + 2x^2 = x^{\frac{1}{3}} + 2x^2$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + 4x$$

$$= \frac{1}{3x^{2/3}} + 4x$$

$$= \frac{1}{3\sqrt[3]{x^2}} + 4x$$

Minds On

Whiteboard Derivatives

$$f(x) = \frac{1}{x^1} = x^{-1}$$

$$f'(x) = -1x^{-2} = \frac{-1}{x^2}$$

Minds On

Try on your own...

2.3 The Product Rule

Let p(x) = f(x)g(x), where f(x) = (x^2 + 2) and g(x) = (x + 5). Show that p'(x) ≠ f'(x)g'(x)

$p(x) = (x^2+2)(x+5)$

$p(x) = x^3 + 5x^2 + 2x + 10$

$p'(x) = 3x^2 + 10x + 2$

$f'(x) \times g'(x) = 2x \cdot 1$

$= 2x$

Action

The Product Rule: If h(x) = f(x)g(x), then h′(x) = f′(x)g(x) + f(x)g′(x)

"The derivative of the product of two functions is equal to the derivative of the first function times the second function plus the first function times the derivative of the second function."

Important to note: In some cases, it is easier to expand and simplify the product before differentiating, rather than using the product rule. (we did this yesterday)

Action

Example 1: Differentiate $h(x) = (x^2 - 3x)(x^5 + 2)$ using the product rule.

$$h(x) = (x^2 - 3x)(x^5 + 2)$$

$$h'(x) = \frac{d}{dx}[(x^2 - 3x)] \cdot (x^5 + 2) + (x^2 - 3x) \cdot \frac{d}{dx}[(x^5 + 2)]$$

$$h'(x) = (2x - 3)(x^5 + 2) + (x^2 - 3x)(5x^4)$$

$$= 2x^6 + 4x - 3x^5 - 6 + 5x^6 - 15x^5$$

$$= 7x^6 - 18x^5 + 4x - 6$$

Action

If we are evaluating the derivative at a specific point, it is sometimes easier to sub this value in before *simplifying* the derivative.

Example 3: Find the value of h`(-1) for the function
$h(x) = (5x^3 + 7x^2 + 3)(2x^2 + x + 6)$

you may skip this step

$$h'(x) = \frac{d}{dx}[(5x^3+7x^2+3)] \cdot (2x^2+x+6) + (5x^3+7x^2+3) \cdot \frac{d}{dx}[(2x^2+x+6)]$$

$$h'(x) = (15x^2+14x)(2x^2+x+6) + (5x^3+7x^2+3)(4x+1)$$

$$h'(-1) = -6$$

Action

Example 4: Find an expression for p`(x) if p(x) = f(x)g(x)h(x).

1. Consider f(x)g(x) to be one function

$$p(x) = [f(x)g(x)]h(x)$$

$$p'(x) = [(f(x)g(x))]' \cdot h(x) + [f(x)g(x)] \cdot h'(x)$$

$$= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

Action

The Power Rule: If $f(x) = [g(x)]^n$, then $f'(x) = n[g(x)]^{n-1}g'(x)$

Action

Example 5: For $h(x) = (x^2 + 3x + 5)^6$, find h`(x).

$$h(x) = (x^2+3x+5)^6$$

$$h'(x) = 6(x^2+3x+5)^5(2x+3)$$

Action

Example 6: Differentiate the rational function $f(x) = \frac{2x+5}{3x-1}$ by first expressing it as a product and then using the product rule.

$f(x) = (3x-1)^{-1}$

$f'(x) = (-1)(3x-1)^{-2}(3)$

$$f(x) = (2x+5)(3x-1)^{-1}$$

$$f'(x) = 2(3x-1)^{-1} + (2x+5)(-1)(3x-1)^{-2}(3)$$

$$f'(x) = \frac{2}{(3x-1)} - \frac{3(2x+5)}{(3x-1)^2}$$

$$f'(x) = \frac{2(3x-1) - 3(2x+5)}{(3x-1)^2}$$

quotient rule

don't need to know

$$f'(x) = \frac{\cancel{6x}-2-\cancel{6x}-15}{(3x-1)^2}$$

$$f'(x) = \frac{-17}{(3x-1)^2}$$

$(6-3t)^4$

Action

Example 7: The position, s, in centimetres, of an object moving in a straight line is given by:
s = t(6 – 3t)^4, t ≥ 0, where t is the time in seconds.
Determine the object`s velocity at t = 2.

roc / derivative

$$v = \frac{ds}{dt} = (1)(6-3t)^4 + (t)(4)(6-3t)^3(-3)$$

$$= (6-3t)^4 - 12t(6-3t)^3$$

at $t=2$, $v = 0$ cm/s

Consolidation

Section 2.3

Look over the questions in section 2.3

Be sure you can do the basics.

Then, look at the application problems, if you know how to do a problem at first glance, solve it if you like.

If you look at a problem and aren't sure what to do, try it out.