Learning Goal: I will be able to use the chain rule to determine derivatives of composite functions.

Minds On: Composite functions reminder

Action: Note and examples

Consolidation: Exit Question

Minds On

Three Question Warm-Up

Differentiate each function given below.

$$f(x) = (x^3 - 2x - 1)(3x^5)$$

$$g(x) = (x^3 - 2x - 1)^5$$

$$h(x) = \frac{(x^3 - 2x)}{(4x - 3)^2}$$

Minds On

Three Question Warm-Up

Differentiate each function given below.

$$f(x) = (x^3 - 2x - 1)(3x^5)$$

$$f'(x) = \frac{d}{dx}(x^3-2x-1)(3x^5) + (x^3-2x-1)\frac{d}{dx}(3x^5)$$

$$= (3x^2-2)(3x^5) + (x^3-2x-1)(15x^4)$$

Should expand

Minds On

Three Question Warm-Up

Differentiate each function given below.

$$g(x) = (x^3 - 2x - 1)^5$$

$$g'(x) = 5(x^3 - 2x - 1)^4(3x^2 - 2)$$

Minds On

Three Question Warm-Up

Differentiate each function given below.

$$h(x) = \frac{(x^3 - 2x)}{(4x - 3)^2}$$

$$h(x) = (x^3 - 2x)(4x - 3)^{-2}$$

$$h'(x) = (3x^2 - 2)(4x - 3)^{-2} + (x^3 - 2x)(-2)(4x - 3)^{-3}(4)$$

Minds On

Reminders

Take Home Assignment

Open Book Test

Minds On

2.5 The Derivatives of Composite Functions

Example 1: If $f(x) = \sqrt{x}$ and $g(x) = x + 5$, find each of the following values:

a) $f(g(4))$

$= \sqrt{(4)+5}$

$= \sqrt{9}$

$= 3$

b) $g(f(4))$

$= \sqrt{4} + 5$

$= 2+5$

$= 7$

c) $f(g(x))$

$= \sqrt{x+5}$

d) $g(f(x))$

$= \sqrt{x} + 5$

The chain rule states how to compute the derivative of the composite function $h(x) = f(g(x))$ in terms of the derivatives of f and g.

The Chain Rule

If f and g are functions that have derivatives, then the composite function $h(x) = f(g(x))$ has a derivative given by $h'(x) = f'(g(x))g'(x)$.

In Leibniz notation

If y is a function of u and u is a function of x (so that y is a composite function), then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, provided that $\frac{dy}{du}$ and $\frac{du}{dx}$ exist.

Action

Example 2: Differentiate $h(x) = (x^2 + x)^{3/2}$

$(u)^{3/2}$

$h(x) = (x^2+x)^{3/2}$

$h'(x) = \frac{3}{2}(x^2+x)^{\frac{1}{2}} \times (2x+1)$

$= \frac{3\sqrt{x^2+x}\,(2x+1)}{2}$

$= \frac{(6x+3)(\sqrt{x^2+x})}{2}$

Action

Example 3: If y = u³ – 2u + 1, where u = 2√x, find $\frac{dy}{dx}$ at x = 4. $\rightarrow 2x^{\frac{1}{2}}$

$$\frac{dy}{du} \times \frac{du}{dx} = (3u^2 - 2)\left(\cancel{\tfrac{1}{2}}\right)\left(\cancel{2}\right)(x)^{-\frac{1}{2}}$$

$$= \frac{3u^2 - 2}{\sqrt{x}}$$

when $x = 4$

$u = 2\sqrt{4}$

$u = 4$

at $x = 4$, $\frac{dy}{dy} = \frac{3(4)^2 - 2}{\sqrt{4}}$

$$= \frac{46}{2}$$

$$= 23$$

Action

Example 4: An environmental study of a certain suburban community suggests that the average daily level of carbon monoxide in the air can be modelled by the function $C(p) = \sqrt{0.5p^2 + 17}$, where C(p) is in parts per million and population p is expressed thousands. It is estimated that t years from now, the population of the community will be p(t) = $3.1 + 0.1t^2$ thousand. At what rate will the carbon monoxide level be changing with respect to time three years from now?

$$\frac{dC}{dt} = \frac{dC}{dp} \times \frac{dp}{dt}$$

$$C(p) = (0.5p^2 + 17)^{\frac{1}{2}}$$

$$= \frac{1}{2}(0.5p^2 + 17)^{-\frac{1}{2}}(p) \times (0.2t)$$

$$\frac{dC}{dt} = \frac{0.2pt}{2\sqrt{0.5p^2+17}}$$

when $t = 3$

$$p(t) = 3.1 + 0.1(3)^2$$
$$= 4$$

when $t = 3$

$$\frac{dC}{dt} = \frac{0.2(4)(3)}{2\sqrt{0.5(4)^2+17}}$$

$$= \frac{2.4}{2\sqrt{25}}$$

$$= 0.24 \text{ ppm/year}$$

Action

Example 5: If y = $(x^2 - 5)^7$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 7(x^2-5)^6(2x)$$

$$= 14x(x^2-5)^6$$

Action

Example 6: Differentiate $h(x) = (x^2 + 3)^4(4x - 5)^3$. Express your answer in a simplified factored form.

$$h'(x) = \frac{d}{dx}\left[(x^2+3)^4\right] \times (4x-5)^3 + (x^2+3)^4 \times \frac{d}{dx}\left[(4x-5)^3\right]$$

$$= 4(x^2+3)^3(2x)(4x-5)^3 + (x^2+3)^4(3)(4x-5)^2(4)$$

$$= 8x(x^2+3)^3(4x-5)^3 + 12(x^2+3)^4(4x-5)^2$$

$$= 4(x^2+3)^3(4x-5)^2\left(2x(4x-5) + 3(x^2+3)\right)$$

$$= 4(x^2+3)^3(4x-5)^2(8x^2 - 10x + 3x^2 + 9)$$

$$= 4(x^2+3)^3(4x-5)^2(11x^2 - 10x + 9)$$

Consolidation

Example 7: Determine the derivative of $g(x) = \left(\frac{1+x^2}{1-x^2}\right)^{10}$

$$g(x) = (1+x^2)^{10}(1-x^2)^{-10}$$

$$g'(x) = 10(1+x^2)^9(2x)(1-x^2)^{-10} + (1+x^2)^{10}(-10)(1-x^2)^{-11}(-2x)$$

$$g'(x) = \frac{20x(1+x^2)^9}{(1-x^2)^{10}} + \frac{20x(1+x^2)^{10}}{(1-x^2)^{11}}$$

* common denominator

$$g'(x) = \frac{20x(1+x^2)^9(1-x^2) + 20x(1+x^2)^{10}}{(1-x^2)^{11}}$$

Factor

$$g'(x) = \frac{20x(1+x^2)^9\left((1\cancel{-x^2})+(1\cancel{+x^2})\right)}{(1-x^2)^{11}}$$

$$g'(x) = \frac{20x(1+x^2)^9(2)}{(1-x^2)^{11}}$$

$$g'(x) = \frac{40x(1+x^2)}{(1-x^2)^{11}}$$

Practice

Section 2.5

Look through the questions in the section. If you know how to do it immediately and aren't worried about the algebra, don't waste your time!

If you find that you don't know exactly how to approach it right away, maybe try it out.

***Bring questions from the textbook that you would like me to go over on Monday.**