4.2 Critical Points, Local Maxima, and Local Minima

A **critical number** is a number, c, in the domain of f(x) such that f'(c) = 0 or f'(c) is undefined. As a result, (c, f(c)) is called a critical point and usually corresponds to local or absolute extrema.

The First Derivative Test

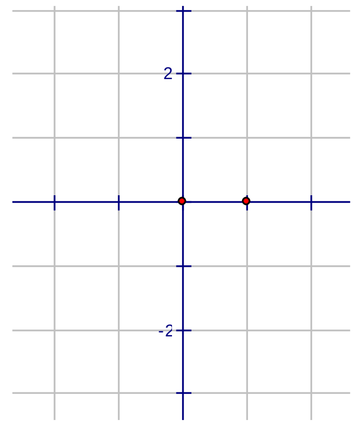
Let c be a critical number of a function f. When moving through x-values from left to right:

- If f'(x) changes from negative to positive at c, then (c, f(c)) is a local minimum of f.
- If f'(x) changes from positive to negative at c, then (c, ,f(c)) is a local maximum of f.
- If f'(x) does not change its sign at c, then (c, f(c)) is neither a local min nor a local max.

Remember from Chapter #2 that derivatives don't exist at cusps and corners on a function's graph. These are also considered extrema, and f'(c) = 0 at these points.

Example 1: For the function $y = x^4 - 8x^3 + 18x^2$, determine all the critical numbers. Determine whether each of these values of x gives a local max, a local min, or neither for the function.

Example 2: Determine whether or not the function $f(x) = x^3$ has a max or min at $(c, f(c))$, where $f'(c) = 0$.



Example 3: For the function $f(x) = (x + 2)^{2/3}$, determine the critical numbers. Use technology to sketch a graph of the function.

Example 4: Given the graph of a polynomial function y = f(x), graph y = f′(x).