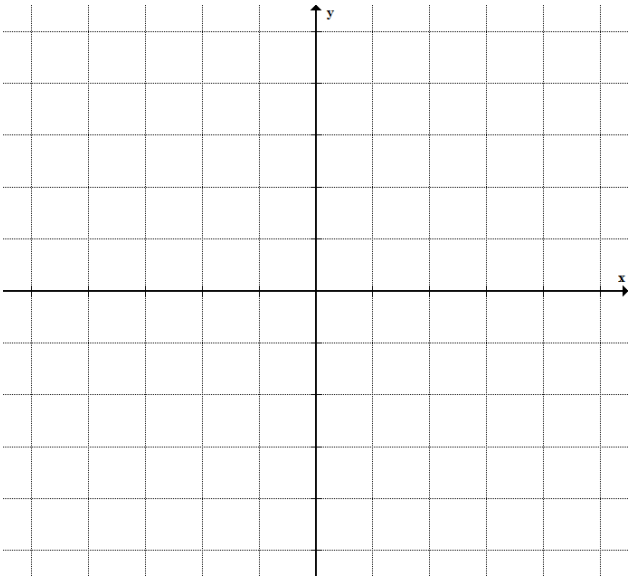
4.3 Vertical and Horizontal Asymptotes

Vertical Asymptotes and Rational Functions

A rational function of the form $f(x) = \frac{p(x)}{q(x)}$ has a vertical asymptote x = c if q(c) = 0 and p(c) ≠ 0.

Example 1: Determine any vertical asymptotes of the function $f(x) = \frac{x}{x^2 + x - 2}$, and describe the behaviour of the graph of the function for values of x near the asymptotes.

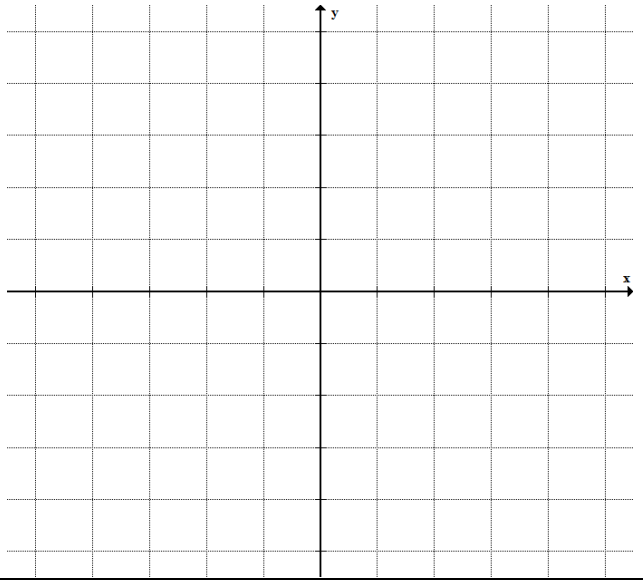


Asymptotes and Infinite Limits

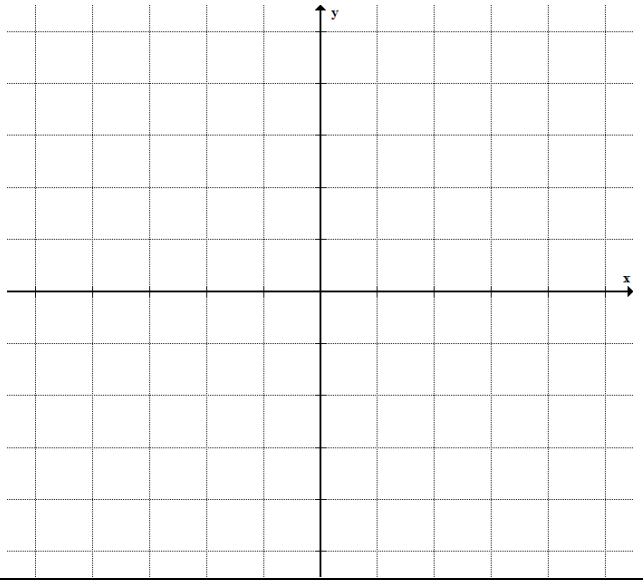
The graph of f(x) has a vertical asymptote, $x = c$, if one of the following infinite limit statements is true: $\lim_{x \to c-} f(x) = +\infty$, $\lim_{x \to c-} f(x) = -\infty$, $\lim_{x \to c+} f(x) = +\infty$, $\lim_{x \to c+} f(x) = -\infty$,

What's does it look like?

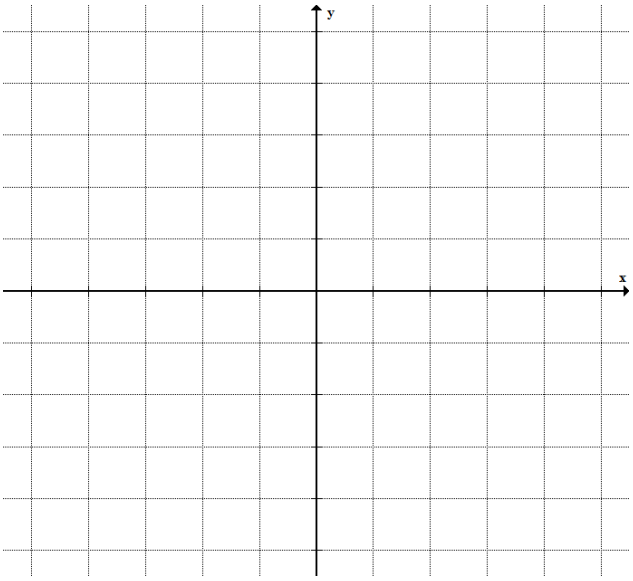
$\lim_{x \to c-} f(x) = +\infty$



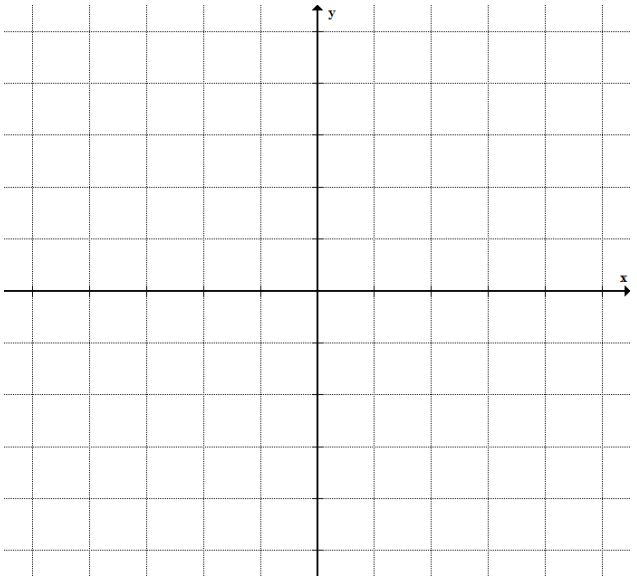
$\lim_{x \to c-} f(x) = -\infty$



$\lim_{x \to c+} f(x) = +\infty$



$\lim_{x \to c+} f(x) = -\infty$



Horizontal Asymptotes and Rational Functions

The Reciprocal Function and Limits at Infinity:	$\lim_{x\to+\infty}\frac{1}{x}=0$ and $\lim_{x\to-\infty}\frac{1}{x}=0$

Example 2: Write each function so the term of highest degree is a factor.

a) $p(x) = x^2 + 4x + 1$

b) $q(x) = 3x^2 - 4x + 5$

What is the value of writing a polynomial in this form?

Why do we care about x becoming large?

Example 3: Determine the value of each of the following:

a) $\lim_{x\to+\infty}\frac{2x-3}{x+1}$

b) $\lim_{x\to-\infty}\frac{x}{x^2+1}$

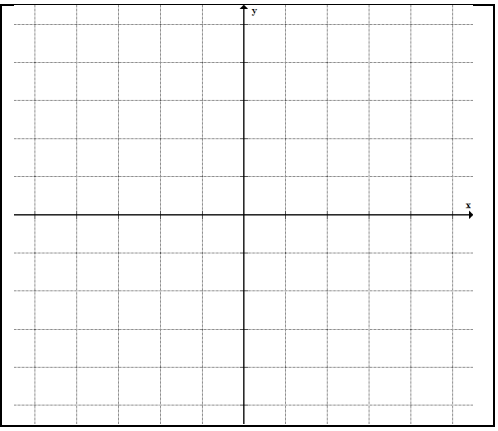
c) $\lim_{x\to+\infty}\frac{2x^2+3}{3x^2-x+4}$

Horizontal Asymptotes and Limits at Infinity

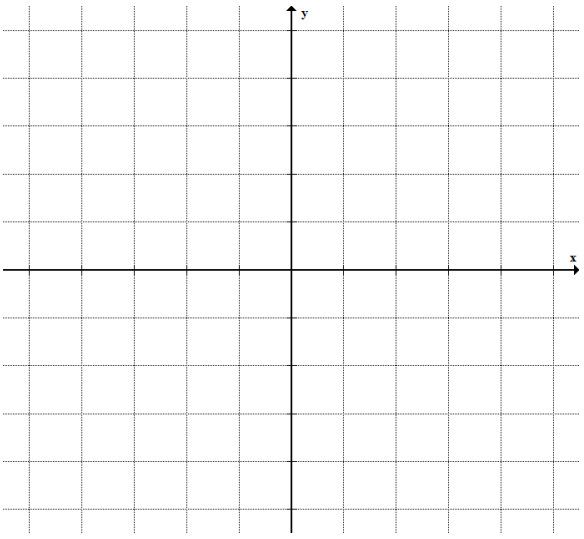
If $\lim_{x\to+\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, we say that the line y = *L* is a horizontal asymptote of the graph of f(x).

For the reciprocal function $f(x) = \frac{1}{x}$:

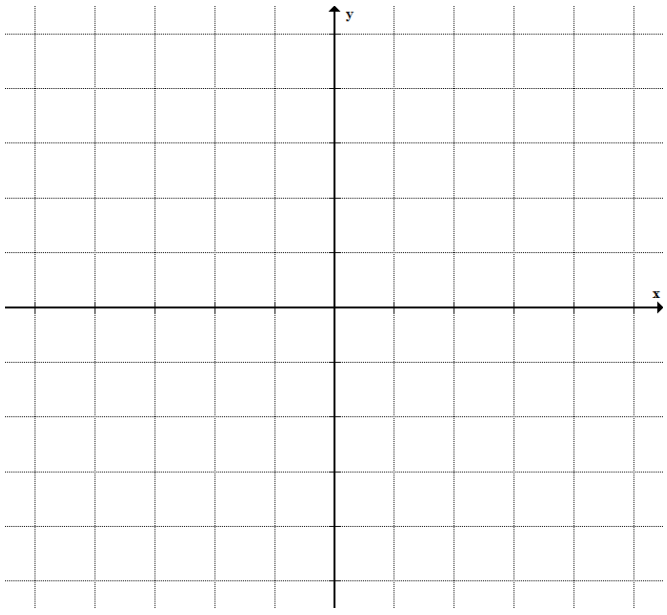
$\lim_{x\to0-} f(x) =$	$\lim_{x\to+\infty} f(x) =$
$\lim_{x\to0+} f(x) =$	$\lim_{x\to-\infty} f(x) =$



Example 4: Determine the equations of any horizontal or vertical asymptotes of the function $f(x) = \frac{3x+5}{2x-1}$. State whether the graph approaches the asymptote from above or below.



Example 5: For the function $f(x) = \frac{3x}{x^2 - x - 6}$, determine the equations of all horizontal or vertical asymptotes. Illustrate the behaviour of the graph as it approaches the asymptotes.



Example 6: Determine the equations of all asymptotes of the graph of $f(x) = \frac{2x^2+3x-1}{x+1}$.