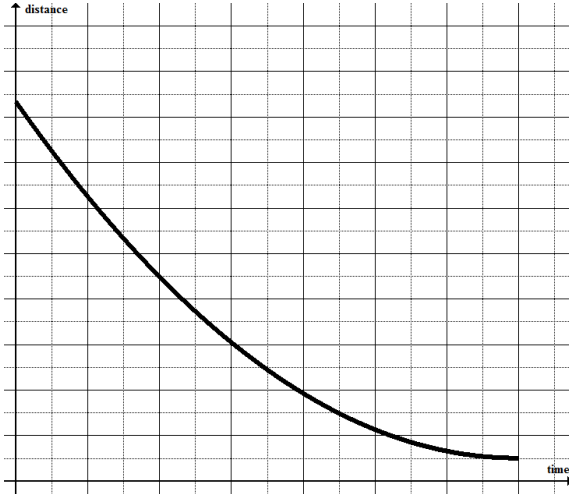


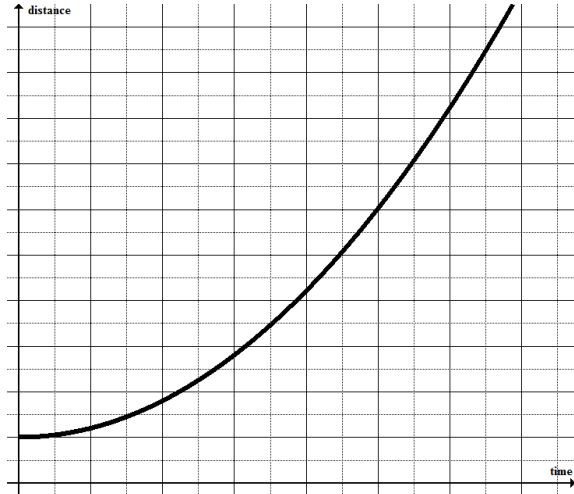
4.4 Concavity and Points of Inflection

The graph of a function $f(x)$ is **concave up** on an interval if $f'(x)$ is increasing on the interval. The graph of a function $f(x)$ is **concave down** on an interval if $f'(x)$ is decreasing on the interval.



Velocity is _____

Acceleration is _____



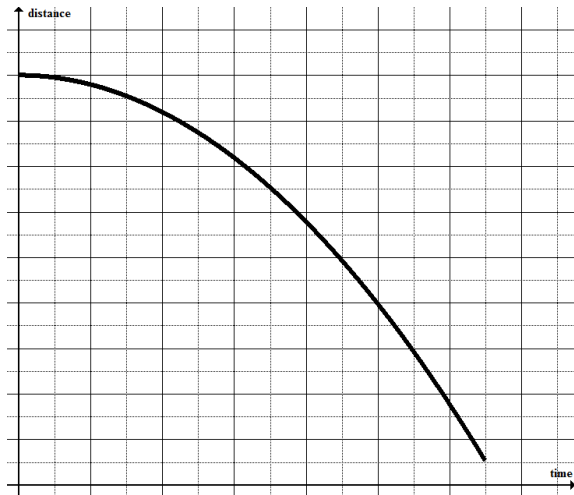
Velocity is _____

Acceleration is _____



Velocity is _____

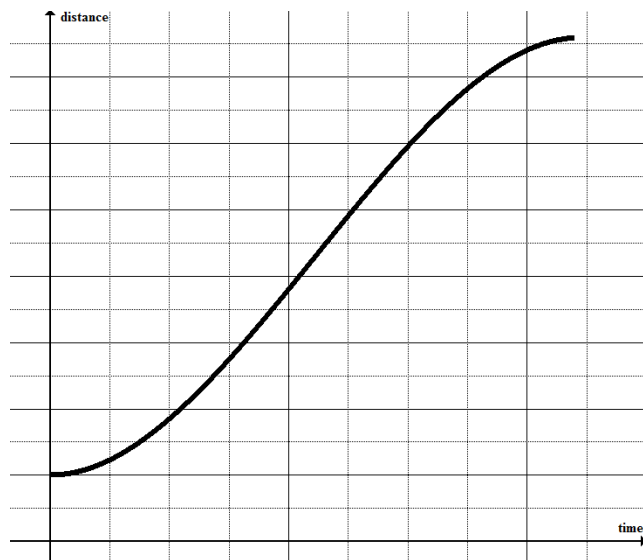
Acceleration is _____



Velocity is _____

Acceleration is _____

A **point of inflection** is a point on the graph of $f(x)$ where the function changes from concave up to concave down, or vice versa.



Identify intervals over which the velocity is increasing and decreasing.

Describe the acceleration over each interval.

Test for Concavity: If $f(x)$ is a differentiable function whose second derivative exists on an open interval I , then

- The graph of $f(x)$ is concave up on I if $f''(x) > 0$ for all values of x in I
- The graph of $f(x)$ is concave down on I if $f''(x) < 0$ for all values of x in I

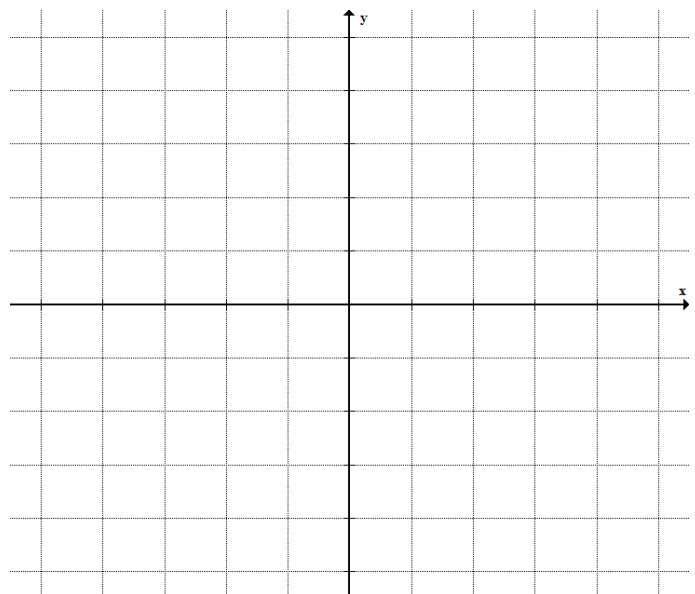
Test for Point of Inflection: If $f(x)$ is a differentiable function whose second derivative exists on an interval containing c then

- The graph of $f(x)$ has a point of inflection where the function changes from concave up to concave down, or vice versa.
- $f''(c) = 0$ or is undefined if $(c, f(c))$ is a point of inflection on the graph of $f(x)$.

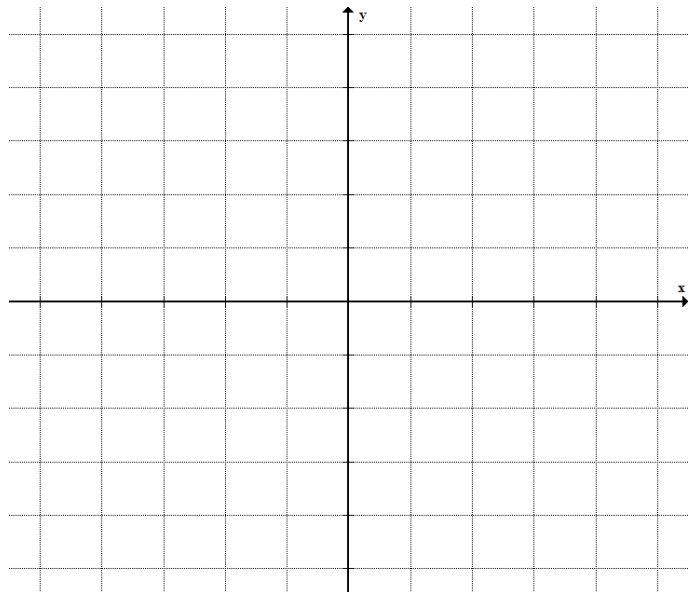
The second derivative test: Suppose that $f(x)$ is a function for which $f'(c) = 0$, and the second derivative of $f(x)$ exists on an interval containing c .

- If $f''(c) > 0$, then $f(c)$ is a local minimum value.
- If $f''(c) < 0$, then $f(c)$ is a local maximum value.
- If $f''(c) = 0$, then the test fails. Use the first derivative test.

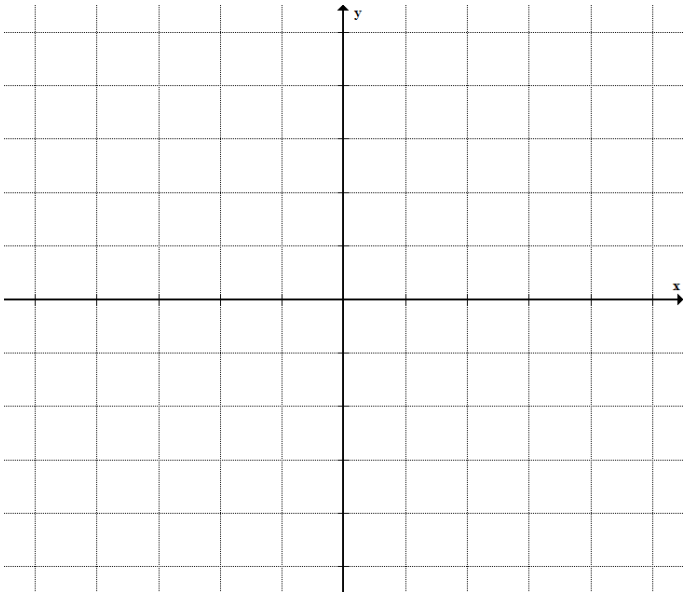
Example 1: Sketch the graph of $y = x^3 - 3x^2 - 9x + 10$



Example 2: Sketch the graph of $f(x) = x^4$



Example 3: Sketch the graph of the function $f(x) = x^{1/3}$



Example 4: Determine any points of inflection on the graph of $f(x) = \frac{1}{x^2+3}$

