

4.5 An Algorithm for Curve Sketching

As each of the following pieces of information are obtained, use them to build the sketch. When using this algorithm, keep two things in mind:

- You will not use all the steps in every situation. Use only the steps that are essential.
- You are familiar with the basic shapes of many functions. Use this knowledge when possible.

An Algorithm for Sketching the Graph of $y = f(x)$

1. Determine any discontinuities or limitations in the domain of $f(x)$. For discontinuities, investigate the function's values on either side of the discontinuity.
2. Determine any vertical asymptotes.
3. Determine any intercepts.
4. Determine any critical numbers by finding where $f'(x) = 0$ or where $f'(x)$ is undefined.
5. Determine the intervals of increase/decrease, and then test critical points to see whether they are local maxima, local minima, or neither.
6. Determine the behaviour of the function for large positive and large negative values of x . This will identify horizontal asymptotes, if they exist. Identify if the function's values approach the horizontal asymptote from above or below.
7. Determine $f''(x)$ and test for points of inflection using the intervals of concavity.
8. Complete the sketch using the above information.

Key Ideas

Use the function, $f(x)$ to:

Determine the domain and any discontinuities; determine the intercepts, and find any asymptotes; and determine function behaviour relative to these asymptotes.

Use the first derivative, $f'(x)$ to:

Find the critical numbers, determine where the function is increasing and where it is decreasing, identify any local maxima or minima.

Use the second derivative, $f''(x)$ to:

Determine where the graph is concave up and where it is concave down, find any points of inflection, and the second derivative can also be used to identify local maxima and minima.