Learning Goal: I will be able to simplify and evaluate limits using the limit properties.

Minds On: What do you notice

Action: Note and examples

Consolidation: Exit Question

RAFT

Minds On

Sketch It!

On your way in, grab a whiteboard, marker and eraser.

Sketch a function with each limit as given below.

$$\lim_{x \to 0} f(x) = 3$$

Minds On

Sketch It!

$$\lim_{x \to 1} g(x) = DNE$$

Minds On

Sketch It!

$$\lim_{x \to 3} h(x) = 2 \text{ and } h(3) \text{ is undefined}$$

Minds On

Sketch It!

$$\lim_{x \to 2^-} m(x) = 4, \lim_{x \to 2^+} m(x) = -2 \text{ and } m(2) = 1$$

Action

What's the Relationship?

Determine each limit.

$\lim\limits_{x \to 2} 3x^2$	$\lim\limits_{x \to 2} 4x$	$\lim\limits_{x \to 2} -1$
$= 12$	$= 8$	$= -1$

$$\lim_{x \to 2}(3x^2 + 4x - 1) = 19$$

Action

Properties of Limits

1. $\lim_{x\to a} k = k$

2. $\lim_{x\to a} x = a$

3. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

4. $\lim_{x\to a}[cf(x)] = c[\lim_{x\to a} f(x)]$, for any constant c

5. $\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]$

6. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided that $\lim_{x\to a} g(x)$ does not equal 0

7. $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$, for any rational number n

If f is a polynomial function, then $\lim_{x\to a} f(x) = f(a)$

Action

Example 2: Evaluate $\lim_{x\to 3}(4x^3 - 2x - 5)$ using the properties of limits

$$\lim_{x\to 3}(4x^3-2x-5)$$

$$= \lim_{x\to 3}(4x^3) + \lim_{x\to 3}(-2x) + \lim_{x\to 3}(-5)$$

$$= 4\left(\lim_{x\to 3}(x)\right)^3 + (-2)\lim_{x\to 3}(x) + \lim_{x\to 3}(-5)$$

$$= 4(3)^3 + (-2)(3) + (-5)$$

$$= 108 - 6 - 5$$

$$= 97$$

Action

Example 3: Evaluate $\lim_{x\to -1} \frac{x^2-5x+2}{2x^3+3x+1}$ using the properties of limits

$$\lim_{x\to -1} \frac{x^2-5x+2}{2x^3+3x+1}$$

$$= \frac{\lim_{x\to -1} x^2-5x+2}{\lim_{x\to -1} 2x^3+3x+1}$$

$$= \frac{(-1)^2-5(-1)+2}{2(-1)^3+3(-1)+1}.$$

$$= \frac{8}{-4}$$

$$= -2$$

Action

Example 4: Evaluate $\lim\limits_{x \to 5} \sqrt{\frac{x^2}{x-1}}$ using the properties of limits

$$= \sqrt{\lim_{x \to 5} \frac{x^2}{x-1}}$$

$$= \sqrt{\frac{\lim_{x \to 5} x^2}{\lim_{x \to 5} x-1}}$$

$$= \sqrt{\frac{25}{4}}$$

$$= \frac{5}{2}$$

RAFT

Action

Sometimes we can't evaluate the limit using direct substitution, because it will result in an ***indeterminate form (0/0).*** In these cases, we find an equivalent fraction that works for all values of f except at x = a.

Example 5: Evaluate $\lim_{x\to 3}\frac{x^2-2x-3}{x-3}$

$$= \lim_{x\to 3}\frac{(x+1)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x\to 3} x+1$$

$$= 4$$

Action

Example 6: Evaluate $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} \times \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}$$

$$= \lim_{x \to 0} \frac{x + 1 - 1}{x(\sqrt{x+1} + 1)}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1} + 1)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+1} + 1}$$

$$= \frac{1}{2}$$

Action

Factoring a Sum or Difference of Cubes

Factoring a Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Factoring a Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Action

Example 7: Evaluate $\lim_{x \to 0} \frac{(x+8)^{\frac{1}{3}} - 2}{x}$ Use substitution to simplify.

Let u represent $(x+8)^{\frac{1}{3}}$ as $x \to 0$, $u \to 2$

then $u^3 = x + 8$ and $x = u^3 - 8$

Our limit becomes $\lim_{u \to 2} \frac{u - 2}{u^3 - 8}$

$= \lim_{u \to 2} \frac{\cancel{u-2}}{\cancel{(u-2)}(u^2 + 2u + 4)}$

$= \lim_{u \to 2} \frac{1}{u^2 + 2u + 4}$

$= \frac{1}{12}$

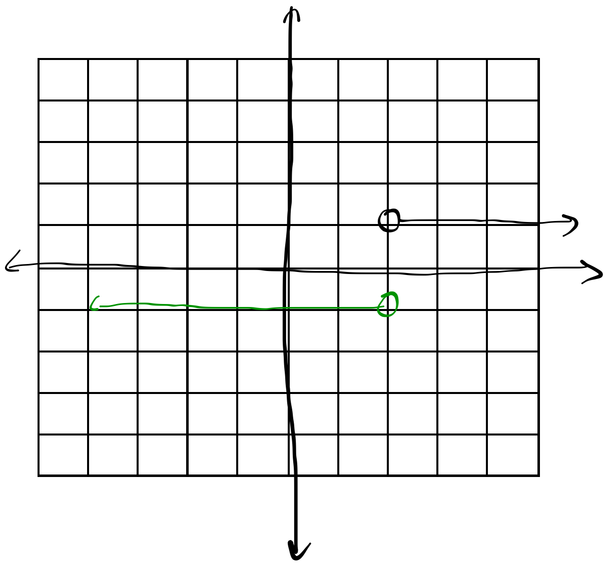
Action

Example 8: Evaluate $\lim_{x \to 2} \frac{|x-2|}{x-2}$. Illustrate with a graph.

left side

as $x \to 2^-$ $f(x) = -1$

as $x \to 2^+$ $f(x) = 1$



$\lim_{x \to 2^-} \neq \lim_{x \to 2^+}$

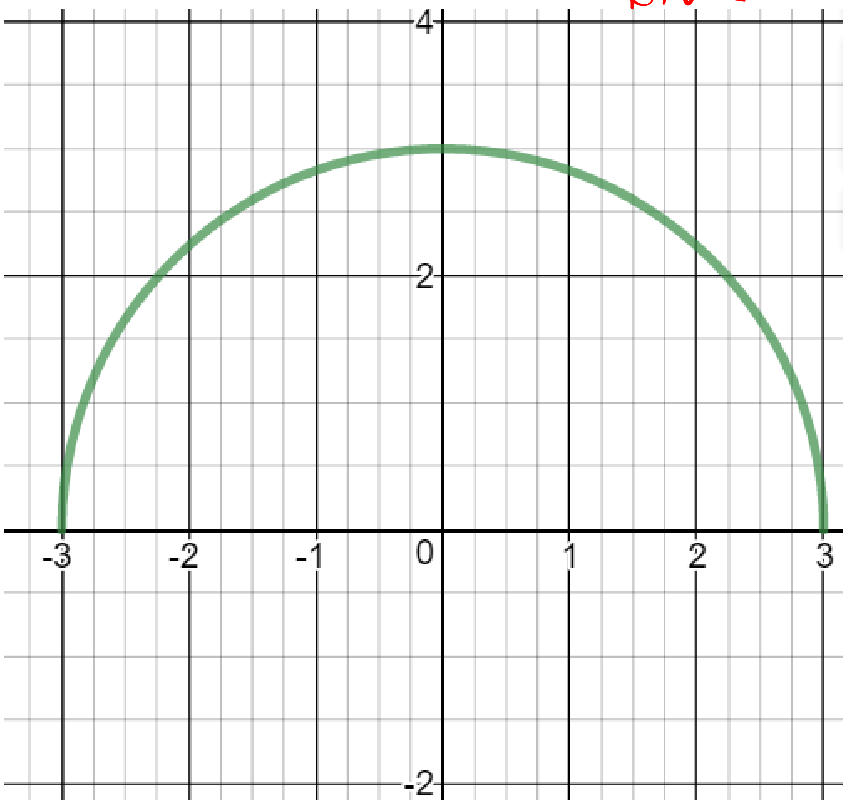
$\therefore \lim_{x \to 2} f(x) = DNE$

Action

Example 9: a) Evaluate $\lim_{x\to 3^-} \sqrt{9-x^2}$. You may use a graphing technique.
$= 0$

b) Explain why the limit as x approaches 3+ cannot be determined.
because the function doesn't exist when x>3

c) What can you conclude about $\lim_{x\to 3} \sqrt{9-x^2}$?
DNE



Consolidation

Practice

Pg. 45

3, 4, 7, **8**, 9