Learning Goal: I will be able to determine and interpret extreme values over intervals.

Minds On: What's my max? What's my min?

Action: Class note + practice

Consolidation: Exit Question

Minds On

What's the max? What's the min?

How do you think you could get a maximum or minimum value of a function over a specific interval?

Use words and/or pictures to support your response.

Determine the maximum value and the minimum value of **$f(x) = 3x + 2$** on the interval $-5 \le x \le 5$.

min: -13 max: 17

Determine the maximum value and the minimum value of **$g(x) = x^2 - 8x + 7$** on the interval $-5 \le x \le 5$.

min: -9 max: 72

Determine the maximum value and the minimum value of **$h(x) = 0.5x^3 + 3x^2$** on the interval $-5 \le x \le 5$.

Action

3.2 Max and Min Values...aka Extreme Values

The maximum value of a function occurs at a "peak" or at an endpoint of an interval.

The minimum value of a function occurs at a "valley" or at an endpoint.

At the peaks and valleys of functions, $f'(c) = 0$.

If a function has a derivative at every point in the interval $a \le x \le b$, calculate f(x) at

- All points in the interval $a \le x \le b$, where $f'(x) = 0$
- The endpoints $x = a$ and $x = b$ of the interval

The maximum value of f(x) on the interval $a \le x \le b$ is the largest of these values, and the minimum of f(x) on the interval is the smallest of these values.

Action

Example 1: Find the extreme values of the function

$f(x) = -2x^3 + 9x^2 + 4$ on the interval $x \in [-1, 5]$.

1. Find peaks & valleys

$f'(x) = -6x^2 + 18x$

$0 = -6x(x-3)$

We have a peak or valley when $x = 0, 3$.

2. Test peaks, valleys and endpoints.

$f(0) = 4$

$f(3) = 31$

$f(-1) = 15$

$f(5) = -21$

$\therefore$ min is -21 when $x = 5$

max is 31 when $x = 3$

Action

Example 2: The amount of current, in amperes (A), in an electrical system is given by the function

$C(t) = -t^3 + t^2 + 21t$, where t is the time in seconds and $0 \leq t \leq 5$. Determine the times at which the current is at its maxim and minimum and determine the amount of current in the system at these times.

1. Find peaks & valleys

$C'(t) = -3t^2 + 2t + 21$

$0 = -3t^2 + 2t + 21$

$0 = -3t^2 + 9t - 7t + 21$

$0 = -3t(t-3) - 7(t-3)$

$0 = (-3t-7)(t-3)$

We have peaks or valleys when $t = \cancel{-\frac{7}{3}}, 3$

2. Test peaks, valleys and end points

$C(0) = 0$

$C(3) = 45$

$C(5) = 5$

$\therefore$ minimum current is 0A when $t = 0$.
maximum current is 45A when $t = 3$.

Action

Example 3: The amount of light intensity on a point is given by the function $l(t) = \frac{t^2 + 2t + 16}{t + 2}$, where t is the time in seconds and t ∈ [0, 14]. Determine the time of minimal intensity.

* We note that this function has an asymptote when $t = -2$. However, -2 is not in our interval.

Step 1: find peaks and valleys

$$l(t) = (t^2 + 2t + 16)(t+2)^{-1}$$

$$l'(t) = (2t+2)(t+2)^{-1} + (t^2+2t+16)(-1)(t+2)^{-2}(1)$$

$$l'(t) = \frac{(t+2)}{(t+2)}\frac{2t+2}{t+2} - \frac{t^2+2t+16}{(t+2)^2}$$

$$l'(t) = \frac{(t+2)(2t+2) - (t^2+2t+16)}{(t+2)^2}$$

$$l'(t) = \frac{2t^2+6t+4-t^2-2t-16}{(t+2)^2}$$

$$l'(t) = \frac{t^2+4t-12}{(t+2)^2}$$

$$0 = \frac{t^2+4t-12}{(t+2)^2}$$

$$0 = \frac{(t+6)(t-2)}{(t+2)^2}$$

$$0 = (t+6)(t-2)$$

$\therefore$ we have peaks or valleys when $t = \cancel{-6}, 2$

not in interval!

2. Test peaks, valleys and endpoints

$l(0) = 8$

$l(2) = 6$

$l(14) = 15$

The minimal intensity happens after 6 seconds.

Consolidation

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