Learning Goal: I will be able to solve optimization problems.

Minds On: Find the max/min

Action: Class note + practice

Consolidation: Exit Question

Minds On

Max. and Min.?

$$f(x) = \frac{-3x + 6}{x - 9}$$

asymptote when x = 9

Determine the maximum and minimum values of f(x) on the interval $0 \le x \le 10$.

9 is in the interval so the max/min are undefined

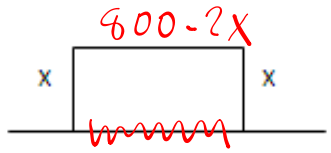
Action

3.3 Optimization

In an optimization problem, you must determine the maximum or minimum value of a quantity. set derivative = 0

An optimization problem can be solved using a mathematical model that is developed using information given in the problem. The numerical solution represents the extreme value of the model.

Example 1: A farmer has 800 m of fencing and wished to enclose a rectangular field. One side of the field is against a country road that is already fenced, so the farmer needs to fence only the remaining three sides of the field. The farmer wants to enclose the maximum possible area and to use all the fencing. How does the farmer determine the dimensions to achieve this goal?



$A(x) = (x)(800-2x)$

$A(x) = 800x - 2x^2$

$A'(x) = 800 - 4x$

When $A'(x) = 0$ $\quad 0 = 800 - 4x$

$x = 200m$

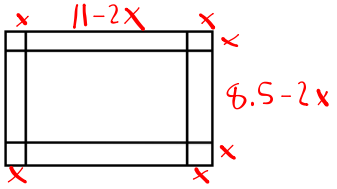
∴ the dimensions are $400m \times 200m$.

the area is $A(x) = (200)(800-2(200))$

$= 80,000 m^2$

Action

Example 2: A piece of paper, 8.5" by 11", is to be used to make a rectangular box with an open top. Determine the dimensions that will give the box with the largest volume.



$V(x) = (11-2x)(8.5-2x)(x)$

$V(x) = 4x^3 - 39x^2 + 93.5x$

$V'(x) = 12x^2 - 78x + 93.5$

Find the limits of x

$x > 0$

$8.5 - 2x > 0$

$-2x > -8.5$

$x < 4.25$

set $V'(x) = 0$

$0 = 12x^2 - 78x + 93.5$

Use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{78 \pm \sqrt{(-78)^2 - 4(12)(93.5)}}{2(12)}$$

$x = 1.59"$ or $x = 4.91"$ impossible

We need to cut out 1.59" x 1.59" squares

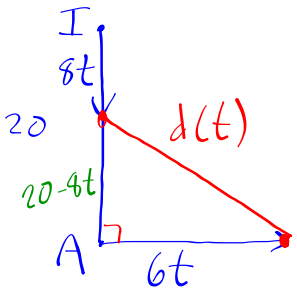
Length $= 11 - 2(1.59) = 7.82$

Width $= 8.5 - 2(1.59) = 5.32$

height $= 1.59$

Action

Example 3: Ian and Ada are both training for a marathon. Ian's house is located 20 km north of Ada's house. At 9:00 a.m. one Saturday, Ian leaves his house and jogs south at 8 km per hour. At the same time, Ada leaves her house and jogs east at 6 km per hour. When are Ian and Ada closest together, given that they both run for 2.5 h?



$$d^2 = (20-4t)^2 + (6t)^2$$

$$d = \sqrt{(20-4t)^2 + (6t)^2}$$

$$d = \sqrt{400 - 160t - 160t + 64t^2 + 36t^2}$$

$$d = \sqrt{100t^2 - 320t + 400}$$

$$d = (100t^2 - 320t + 400)^{\frac{1}{2}}$$

$$d' = \frac{1}{2}(100t^2 - 320t + 400)^{-\frac{1}{2}}(200t - 320)$$

$$d' = \frac{200t - 320}{2\sqrt{100t^2 - 320t + 400}}$$

$$0 = \frac{200t - 320}{2\sqrt{100t^2 - 320t + 460}}$$

$0 = 200t - 320$

$t = 1.6$

Test

$d(0) = \sqrt{(20 - 6(0))^2 + (6(0))^2} = 20$

$d(1.6) = \sqrt{(20 - 6(1.6))^2 + (6(1.6))^2} = 12$

$d(2.5) = \sqrt{(20 - 6(2.5))^2 + (6(2.5))^2} = 15$

∴ they are closest after 1.6 hours at 12 km apart.

time is: 10:36 am.

Consolidation

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