MINDS ON

Student Council sells *x* bags of chips for $1.25 each. They pay $0.75 for the chips.

Write an expression for total revenue.

Write an expression for total profit.

$R(x) = 1.25x$

$P(x) = 1.25x - 0.75x$

$= 0.50x$

ACTION

Example 1: A commuter train carried 2000 passengers daily from a suburb into a large city. The cost to ride the train is \$7 per person. Market research shows that 40 fewer people would ride the train for each \$0.10 increase in the fare, and 40 more people would ride the train for each \$0.10 decrease. If the capacity of the train is 2600 passengers, and carrying fewer than 1600 passengers means costs exceed revenue, what fare should the railway charge to get the largest possible revenue?

Let x represent the # of \$0.10 increases

# of tickets sold: $2000 - 40x$

cost per ticket: $7 + 0.10x$

Determine the bounds on x

$2000 - 40x \leq 2600$

$x \geq -15$

$2000 - 40x \geq 1600$

$x \leq 10$

$R(x) = (2000 - 40x)(7 + 0.10x)$

$$R(x) = -4x^2 - 80x + 14000$$

$$R'(x) = -8x - 80$$

$$0 = -8x - 80$$

$$x = -10$$

$$\boxed{R(-10) = 14{,}400}$$

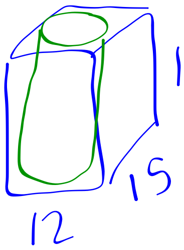
$$R(-15) = 14{,}300$$

$$R(10) = 12{,}800$$

$\therefore$ max revenue is reached with 10 10cent decreases in price. So tickets will be \$6 each.

Example 2: A cylindrical chemical storage tank with a capacity of 1000 m^3 is going to be constructed in a warehouse that is 12 m by 15 m, with a height of 11 m. The specifications call for the base to be made of sheet steel that costs \$100/m^2, the top to me made of sheet steel that costs \$50/m^2, and the walls to be made of sheet steel that costs \$80/m^2.

a) Determine whether it is possible for a tank of this capacity to fit in the warehouse. If it is possible, state the restrictions on the radius.



$V_{cyl} = \pi r^2 h$

biggest cylinder possible

$V_{cyl} = \pi (6)^2 (11)$
$= 1244.07$ ✓ we're good

Restrictions on radius

$r \leq 6$

smallest radius

- when radius is smallest, $h = 11$ m

$V = \pi r^2 h$

$1000 = \pi r^2 (11)$

$r = 5.34$ m

$\therefore 5.34 \leq r \leq 6$

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b) If fitting the tank in the warehouse is possible, determine the proportions that meet the conditions and that minimize the cost of the steel for construction.

cost will depend on surface area

$SA_{cyl} = 2\pi r^2 + 2\pi rh$

try to minimize surface area

we can't have two variables!

rearrange volume of cylinder for h

$V = \pi r^2 h$

$1000 = \pi r^2 h$

$h = \dfrac{1000}{\pi r^2}$

$$SA = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$$

$$SA = 2\pi r^2 + \frac{2000}{r}$$

Cost equation → 3 parts

$$SA = \underbrace{\pi r^2}_{\text{top}} + \underbrace{\pi r^2}_{\text{bottom}} + \underbrace{\frac{2000}{r}}_{\text{walls}}$$

$$\text{cost} = 50\pi r^2 + 100\pi r^2 + 80\left(\frac{2000}{r}\right)$$

$$\boxed{C(r) = 150\pi r^2 + 160000r^{-1}}$$

$$C'(r) = 300\pi r + (-1)(160000)r^{-2}$$

$$C'(r) = 300\pi r - \frac{160000}{r^2}$$

To find max/min set $C'(r)=0$

$$0=300\pi r-\frac{160000}{r^2}$$

$$\frac{160000}{r^2}=300\pi r$$

$$160000=300\pi r^3$$

$$r^3=\frac{160000}{300\pi}$$

$$r=\sqrt[3]{\frac{160000}{300\pi}}$$

$$r=5.54$$

this within our bounds

Test the bounds and our value

$C(5.38) = 43,379.50$

$C(5.54) = 43,343.94$

$C(6) = 43,631.27$

$\therefore r = 5.54 \, m$

$$h = \frac{1000}{\pi(5.54)^2}$$

$$= 10.37 \, m$$

CONSOLIDATION

Section 3.4

Question 9 - Similar to the cylinder one

Question 11a - Similar to the profit one