

Learning Goal: I will be able to calculate derivatives of functions involving $\tan(x)$.

Minds On: Derivative of $\tan(x)$

Action: Class note + practice

Consolidation: Putting it together.



Warm-Up Question

Determine the derivative of $f(x)$

$$f(x) = \frac{\sin x}{\cos x} = \tan x$$

$$f(x) = \sin x (\cos x)^{-1}$$

$$f'(x) = \cos x (\cos x)^{-1} + \sin x (-1)(\cos x)^{-2}$$

$$= \frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

$$= \sec^2 x$$

Action

Derivative of $f(x) = \tan x$

Determine the derivative of $f(x) = \tan x$

$$f'(x) = \sec^2 x$$

Action

Example 1: Determine the derivative of $y = \tan(x^2 + 3x)$

$$\frac{dy}{dx} = \sec^2(x^2 + 3x) (2x + 3)$$

Action

Example 2: Determine the derivative of $y = (\sin x + \tan x)^4$

$$\frac{dy}{dx} = 4(\sin x + \tan x)^3 (\cos x + \sec^2 x)$$

Action

Example 3: Determine the derivative of $y = x \tan(2x - 1)$

$$\frac{dy}{dx} = \tan(2x-1) + x \sec^2(2x-1)(2)$$

$$= \tan(2x-1) + 2x \sec^2(2x-1)$$

Consolidation

Putting it Together

Determine the derivative of $\cot x$ using 2 different algebraic methods.

$$f(x) = \cot x$$

$$f(x) = (\tan x)^{-1}$$

$$f'(x) = -1(\tan x)^{-2} (\sec^2 x)$$

$$= \frac{-\sec^2 x}{\tan^2 x}$$

$$= \frac{-1}{\frac{\cos^2 x}{\sin^2 x}}$$

$$= \frac{-1}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\sin^2 x}$$

$$= -\csc^2 x$$

$$f(x) = (\cos x)(\sin x)^{-1}$$

$$f'(x) = (-\sin x)(\sin x)^{-1} + (\cos x)(-1)(\sin x)^{-2} (\cos x)$$

$$= -1 - \frac{\cos^2 x}{\sin^2 x}$$

$$= -1 - \cot^2 x$$

$$= -\csc^2 x$$