2.1 The Derivative Function

When we were determining slopes of tangents (instantaneous rate of change) in the last unit, we looked at the average rate of change between two points, as the distance, h, between these points approached zero. This limit is a central part of calculus, so let's define it:

The **derivative** of f at the number a is given by: $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$, provided that this limit exists. The notation f'(a) is used to show the derivative of f(x) at x = a.
We pronounce it "f prime of a".

Example 1:

a) Determine the derivative of f(x) = x^2 at an arbitrary value of x.

b) Determine the slopes of the tangents to the parabola y = x^2 at x = -2, 0 and 1.

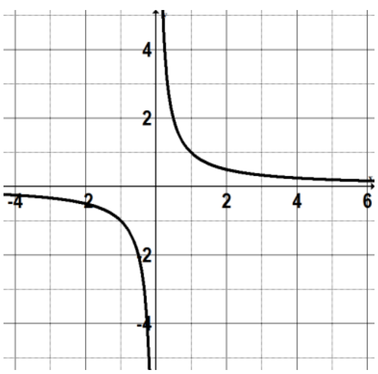
If we use an arbitrary "a" value to get the derivative, we'll get an answer that's a function rather than a numerical answer. This is called the **derivative function**:

The derivative of f(x) with respect to x is the function f'(x), where
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$, provided that this limit exists.

***Important Note (don't forget for your exam): When we use this limit to determine the derivative of a function, it is called <u>determining the derivative from first principles</u>.**

Example 2: Determine an equation of the tangent to the graph of $f(x) = \frac{1}{x}$ at the point x = 2.

Example 3: Determine an equation of the line that is perpendicular to the tangent to the graph of $f(x) = \frac{1}{x}$ at x = 2 and intersects it at the point of tangency.



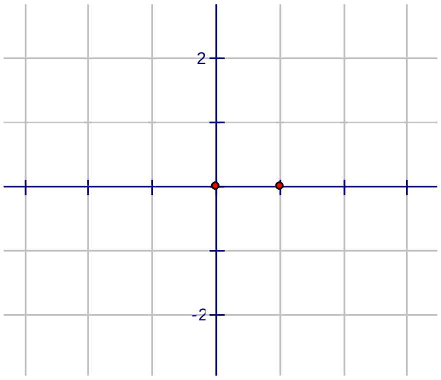
The line we found in example 3 has a proper name:

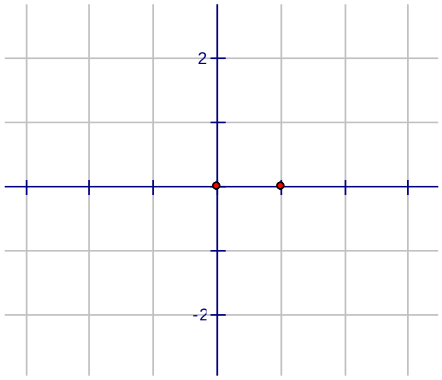
The **normal** to the graph of f at point P is the line that is perpendicular to the tangent at P.

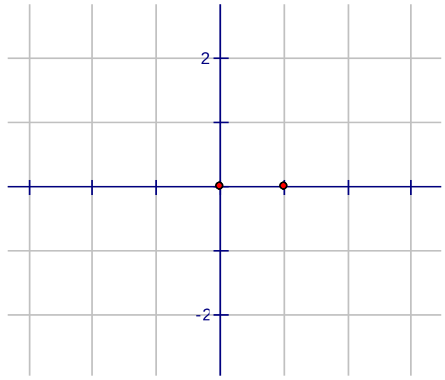
The Existence of Derivatives

We say that a function f is differentiable at a if f'(a) exists. At points where f is not differentiable, we say that the derivative does not exist. There are 3 common ways for a derivative to fail to exist:

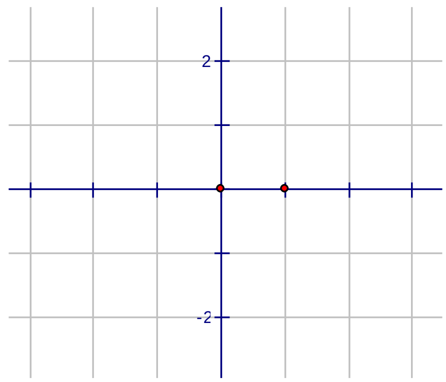
Cusp | Vertical Tangent | Discontinuity







<u>Example 4:</u> Show that the absolute value function $f(x) = |x|$is not differentiable at x = 0.



Other Notation for Derivatives

The most commonly used notation for derivatives if f'(x) or y'. We can also use the symbol $\frac{dy}{dx}$ instead of f'(x). The $\frac{dy}{dx}$ is called Leibniz notation, and is ***not a fraction***.

Exit Question: Determine the derivative f'(t) of the function $f(t) = \sqrt{t}$, t ≥ 0.