New Unit

Polynomial Equations and Inequalities

Minds on

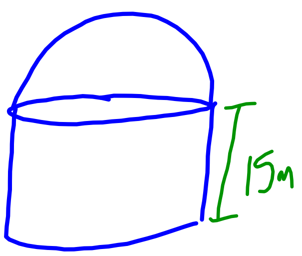
What's that Formula?

A family is planning on building a silo for grain storage. The silo will consist of a cylindrical base with a semi-sphere on top.

If the total volume is to be **684π** m³, and the height of the cylindrical portion is **15** m, what are possible values for the radius of the silo?

$V_{cylinder} = \pi r^2 h$

$V_{sphere} = \frac{4}{3}\pi r^3$



$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$\cancel{684\pi} = \pi r^2(15) + \frac{2}{3}\pi r^3$

$\cancel{-684\pi} \qquad -684\pi$

$0 = 15\pi r^2 + \frac{2}{3}\pi r^3 - 684\pi$

$\pi\left(\underset{\frac{1}{3}}{\underline{\frac{2}{3}r^3}} + \underset{\frac{1}{3}}{\underline{15r^2}} - \underset{\frac{1}{3}}{\underline{684}}\right) = 0$

$\frac{\pi}{3}(2r^3 + 45r^2 - 2052) = 0$

Action!

The Factor Theorem Part II

When dealing with a function with a leading coefficient ≠ 1, numbers that can make $f(x) = 0$ are of the form ***p/q***, where p is a factor of the constant term, and q is a factor of the leading coefficient.

$\frac{\pi}{3}(2r^3+45r^2-2052)=0$

Possible values:

$\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{6}{1}, \frac{9}{1}, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}$

To the TI-83s!

$$\frac{\pi}{3}(2r^3+45r^2-2052)=0$$

r-6 is a factor... r=6 is a possible radius

$$\begin{array}{r|rrrr} 6 & 2 & 45 & 0 & -2052 \\ & & 12 & 342 & 2052 \\ \hline & 2 & 57 & 342 & 0 \end{array}$$

$$\frac{\pi}{3}(r-6)(2r^2+57r+342)=0$$

Use the quadratic formula.

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad r=\frac{-57\pm\sqrt{57^2-4(2)(342)}}{2(2)}$$

$$r=\frac{-57\pm\sqrt{513}}{4}$$

$$r=\frac{-57+\sqrt{513}}{4} \quad \text{or} \quad \frac{-57-\sqrt{513}}{4}$$

$$r=-8.6 \qquad r=-19.9$$

The radius must be r=6.

Action!

Solving a Cubic

Solve $4x^3 - 12x^2 - x + 3 = 0$. $\frac{p}{q}$

Method 1: The factor theorem.

Possible zeros

$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

$x-3$ is a Factor

$$\begin{array}{r|rrrr} 3 & 4 & -12 & -1 & 3 \\ & & 12 & 0 & -3 \\ \hline & 4 & 0 & -1 & 0 \end{array}$$

$$(x-3)(4x^2-1)=0$$

$$(x-3)(2x+1)(2x-1)=0$$

$2x+1=0$
$x=-\frac{1}{2}$

$2x-1=0$
$x=\frac{1}{2}$

$\therefore$ the zeros are $x=3, -\frac{1}{2}, \frac{1}{2}$

Action!

Solving a Cubic

Solve $4x^3 - 12x^2 - x + 3 = 0$.

Method 2: Factoring by Grouping

$$4x^3 - 12x^2 - x + 3 = 0$$

$$4x^2(x-3) - 1(x-3) = 0$$

$$(x-3)(4x^2-1) = 0$$

$$(x-3)(2x+1)(2x-1) = 0$$

$\therefore x = 3, -\frac{1}{2}, \frac{1}{2}$ are solutions

Action!

Solving a Cubic

Solve $4x^3 - 12x^2 - x + 3 = 0$.

Method 3: On the Graphing Calculator

$x = -0.5$ or $-\frac{1}{2}$

$x = 0.5$ or $\frac{1}{2}$

Action!

Where do we meet?

The heights of two orcas above the water in centimetres for a period of 8 seconds can be modelled by the equations:

$h(t) = 20t^3 - 200t^2 + 300t - 200$

$h(t) = 2t^4 - 17t^3 + 27t^2 - 252t + 232$

Determine when the orcas are at the same height over the 8-second period.

$$20t^3 - 200t^2 + 300t - 200 = 2t^4 - 17t^3 + 27t^2 - 252t + 232$$
$$-20t^3 + 200t^2 - 300t + 200 \qquad -20t^3 + 200t^2 - 300t + 200$$

$$2t^4 - 37t^3 + 227t^2 - 552t + 432 = 0$$

$t-4$ is a root

$$\begin{array}{r|rrrrr} 4 & 2 & -37 & 227 & -552 & +432 \\ & & 8 & -116 & 444 & -432 \\ \hline & 2 & -29 & 111 & -108 & 0 \end{array}$$

$$(t-4)(2t^3 - 29t^2 + 111t - 108) = 0$$

$t-9$ is a solution

$$\begin{array}{r|rrrr} 9 & 2 & -29 & 111 & -108 \\ & & 18 & -99 & 108 \\ \hline & 2 & -11 & 12 & 0 \end{array}$$

$$(t-4)(t-9)(2x^2 - 11x + 12) = 0$$
$$(t-4)(t-9)(2x^2 - 8x - 3x + 12) = 0$$
$$(t-4)(t-9)(2t(t-4) - 3(t-4)) = 0$$
$$(t-4)(t-9)(t-4)(2t-3) = 0$$
$$(t-4)^2(t-9)(2t-3) = 0$$

$t = 4$ $\qquad \cancel{t = 9}$ $\qquad 2t - 3 = 0$, $t = \frac{3}{2}$ or 1.5

They are at the same height after 1.5s and 4s.

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2, 6, 7, 8, 16