Learning Goal: I will be able to determine and interpret extreme values over intervals.

Minds On: What's my max? What's my min?

Action: Class note + practice

Consolidation: Exit Question

The position of an object is given by the function **$s(t) = -2t^2 + 8t + 10$**

a) Determine an equation for the velocity and acceleration.
b) Determine when the object is at rest.

a) $v(t) = s'(t) = -4t + 8$
$a(t) = v'(t) = s''(t) = -4$

b) $v(t) = 0$
$-4t + 8 = 0$
$t = 2$

You are going 10 m/s.

If your acceleration is 2 m/s², what is your speed after 2 minutes? (120s)

speed at 0s = 10m/s

Every second, speed ↑ by 2m/s

So, after 120 seconds, speed has increased by 2m/s 120 times. (240 m/s increase)

$$v_f = v_i + at$$
$$= 10 + 2(120)$$
$$= 250 \text{ m/s}$$

Minds On

What's the max? What's the min?

How do you think you could get a maximum or minimum value of a function over a specific interval?
(what are the minimum and maximum values of some function f(x) **between x = -2 and x = 4**?)

Use words and pictures to support your response.

Determine the maximum value and the minimum value of **f(x) = 3x + 2** on the interval $-5 \le x \le 5$. subbed in end points

Determine the maximum value and the minimum value of g**(x) = x² - 8x + 7** on the interval $-5 \le x \le 5$.

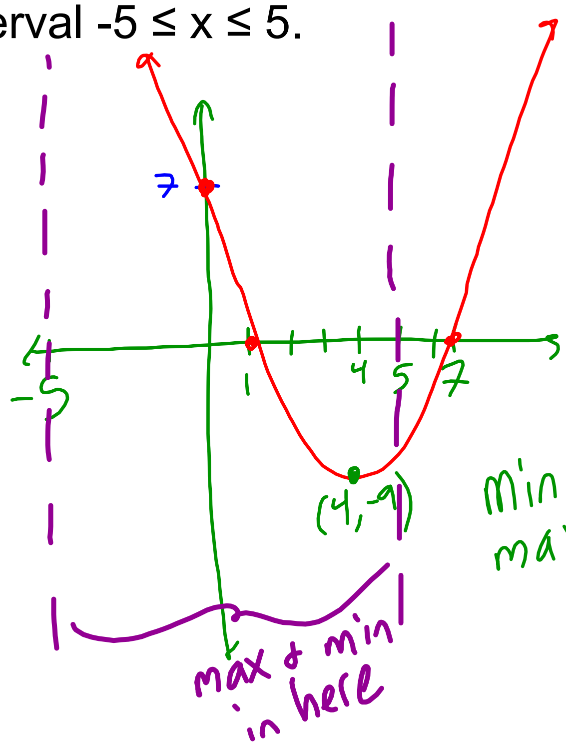
v

Determine the absolute maximum value and the absolute minimum value of $g(x) = x^2 - 8x + 7$ on the interval $-5 \le x \le 5$.

min =

max =

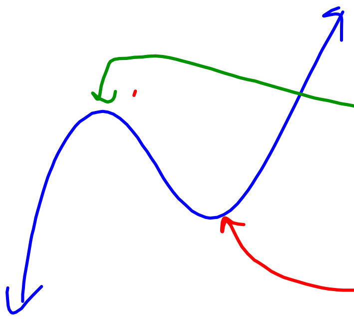
$f(x) = (x-1)(x-7)$



min = -9
max = 72

when x = -5

Determine the maximum value and the minimum value of $h(\mathbf{x}) = \mathbf{0.5x^3 + 3x^2}$ on the interval $-5 \le x \le 5$.



Max is either or when $x=5$ assuming the interval includes the peak & valley.

Min is either or when $x=-5$, assuming the interval includes the peak & valley.

How do we find the peak and the valley?

Peaks and valleys occur when the rate of change is 0 (slope is 0) so the derivative is zero!

Find the derivative, then figure out when it's zero.

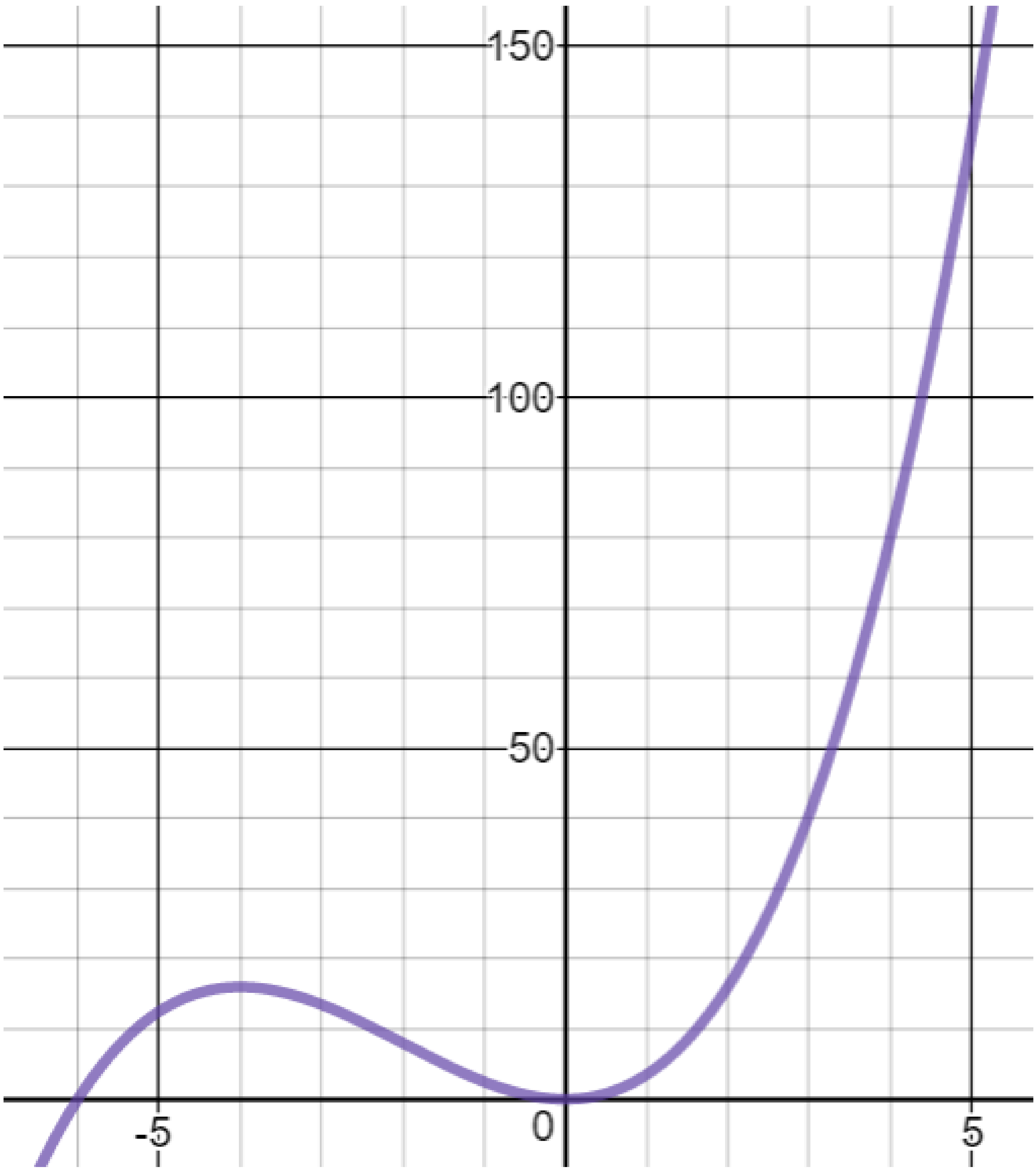
$$h'(x) = 1.5x^2 + 6x$$
$$= 1.5x(x+4)$$

$h'(x) = 0$ when $x = 0, -4$

Now, figure out value of function (y-value) when $x = 0, 4$ and at the ends ($x = -5, 5$)

$h(0) = 0$ min $\quad$ $h(-5) = 12.5$

$h(-4) = 16$ $\quad$ $h(5) = 137.5$ max


150
100
50
-5
0
5

Action

3.3 Max and Min Values...aka Extreme Values

The maximum value of a function occurs at a "peak" or at an endpoint of an interval.

The minimum value of a function occurs at a "valley" or at an endpoint.

At the peaks and valleys of functions, $f'(x) = 0$. (roc = 0 at peak/valley)

If a function has a derivative at every point in the interval $a \le x \le b$, calculate f(x) at

- All points in the interval $a \le x \le b$, where $f'(x) = 0$ (peak/valley)
- The endpoints $x = a$ and $x = b$ of the interval

The maximum value of f(x) on the interval $a \le x \le b$ is the largest of these values, and the minimum of f(x) on the interval is the smallest of these values.

Action

highest and lowest

Example 1: Find the extreme values of the function

$f(x) = -2x^3 + 9x^2 + 4$ on the interval $x \in [-1, 5]$. → $-1 \leq x \leq 5$

Find peaks & valleys

when $f'(x) = 0$

$f'(x) = -6x^2 + 18x$

$-6x^2 + 18x = 0$

$-6x(x-3) = 0$

$x = 0, 3$

Test $f(0)$ & $f(3)$

$f(0) = 4$

$\boxed{f(3) = 31}$ max

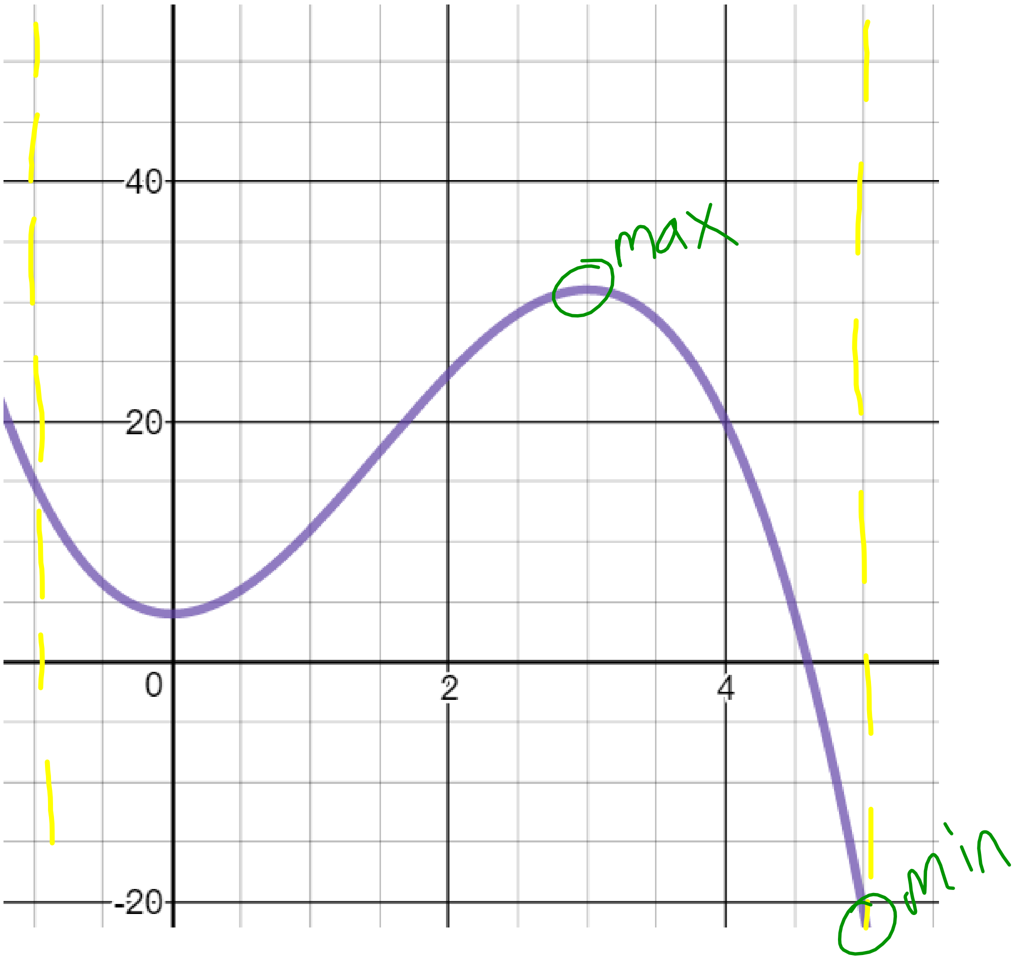
Find function @ end points

$f(-1) = 15$

$\boxed{f(5) = -21}$

max of 31 when $x=3$

min of -21 when $x=5$


40
20
0
2
4
-20
max
min

Action

Example 2: The amount of current, in amperes (A), in an electrical system is given by the function

$C(t) = -t^3 + t^2 + 21t$, where t is the time in seconds and $0 \le t \le 5$. Determine the times at which the current is at its maxim and minimum and determine the amount of current in the system at these times.

Find peaks/valleys

$C'(t) = -3t^2 + 2t + 21$

$-3t^2 + 2t + 21 = 0$

$-3t^2 + \underbrace{9t - 7t}_{} + 21 = 0$

$-3t(t-3) - 7(t-3)$

$(t-3)(-3t-7) = 0$

$t = 3, \cancel{-\frac{7}{3}}$ not in domain

$\boxed{C(3) = 45}$

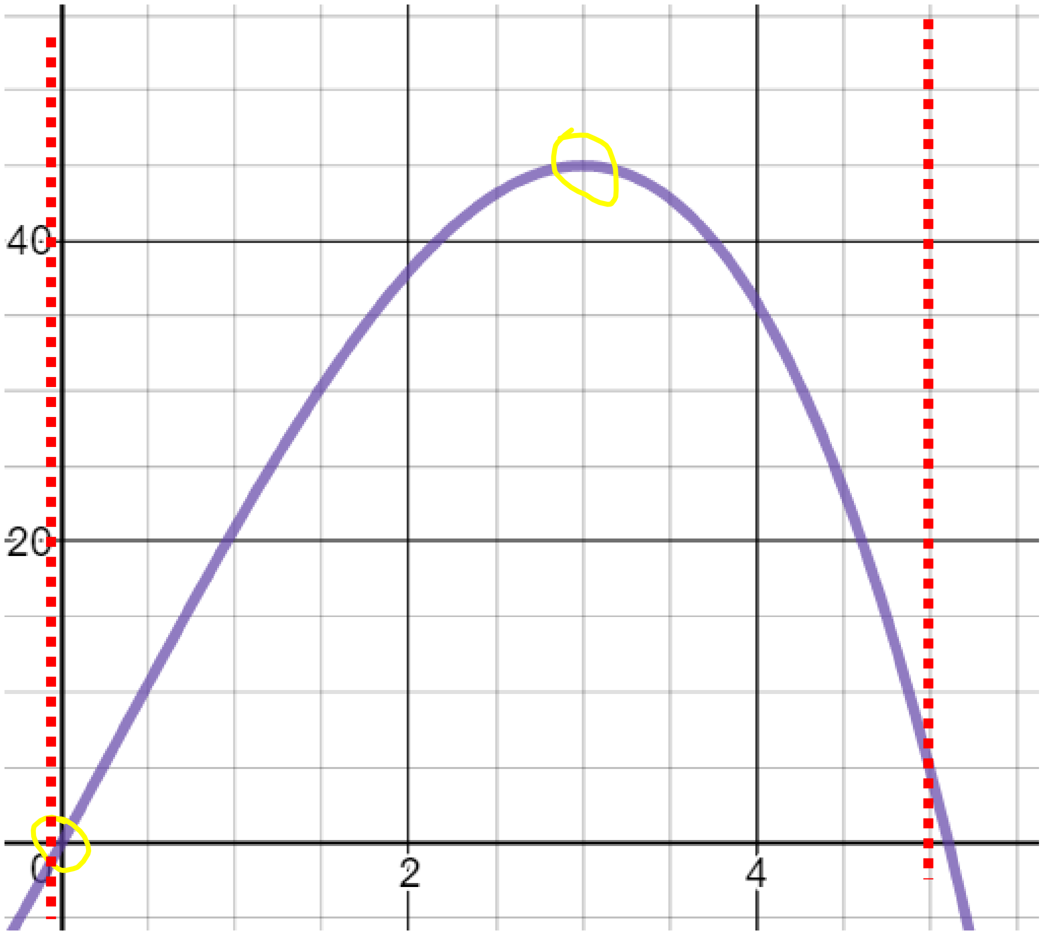
Find function @ end points

$\boxed{C(0) = 0}$

$C(5) = 5$

min of 0A when $t = 0$

max of 45A when $t = 3$


40
20
0
2
4

Action

Example 3: The amount of light intensity on a point is given by the function $l(t) = \frac{t^2+2t+16}{t+2}$, where t is the time in seconds and t ∈ [0, 14]. Determine the time of minimal intensity.

$$l(t) = (t^2+2t+16)(t+2)^{-1}$$

$$l'(t) = (2t+2)(t+2)^{-1} + (t^2+2t+16)(-1)(t+2)^{-2}$$

$$= \frac{2t+2(t+2)}{(t+2)(t+2)} + \frac{-(t^2+2t+16)}{(t+2)^2}$$

$$= \frac{2t^2+4t+2t+4-t^2-2t-16}{(t+2)^2}$$

$$= \frac{t^2+4t-12}{(t+2)^2}$$

peak/valley when $l'(t) = 0$

$$\cancel{(t+2)^2}\,\frac{t^2+4t-12}{\cancel{(t+2)^2}} = \underbrace{0\,(t+2)^2}_{0}$$

$$t^2+4t-12=0$$

$$(t+6)(t-2)=0$$

Test $l(t)$ when $t=-6, 2, 0, 14$

$\cancel{l(-6)} =$ not in domain

$\boxed{l(2)=6}$ min

$l(0) = \cancel{8}$

$\boxed{l(14)=15}$ max

Consolidation

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