What's Going On?

Checking In

Minds on — Identities you know

Action! — Identities you don't

Consolidation — Simplifying and Proving by Factoring

Learning Goal - I will be able to prove trigonometric identities

I dun goofed up...

There were some notation issues with an example I did yesterday, before RAFT, I quickly want to run through them.

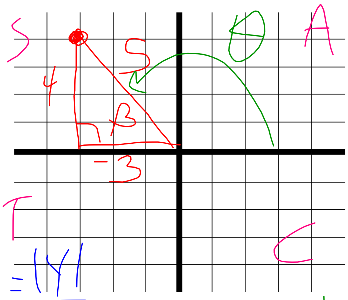
Please copy it down!!

2) Now, choose the point P(-3, 4) on the circumference of the circle.

a) Determine the primary trig raos for the principal angle. $\Rightarrow \theta$

θ is always our principal angle

We will use β as the related acute.



$\sin\beta = \frac{|y|}{r}$ $\cos\beta = \frac{|x|}{r}$ $\tan\beta = \frac{|y|}{|x|}$

* the absolute value symbols are used because we don't care about the sign for the related acute

$\sin\beta = \frac{4}{5}$ $\cos\beta = \frac{3}{5}$ $\tan\beta = \frac{4}{3}$

* We are in Q2 (S) so....

$\boxed{\sin\theta = \sin\beta \quad \cos\theta = -\cos\beta \quad \tan\theta = -\tan\beta}$

$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{-3}{5}$ $\tan\theta = -\frac{4}{3}$

b) Determine the principal angle to the nearest degree.

* We are in Q2, so...

$$\theta = 180 - \beta$$

First find β

$$\sin\beta = \frac{4}{5}$$

$$\beta = \sin^{-1}\left(\frac{4}{5}\right)$$

$$\boxed{\beta = 53^\circ}$$

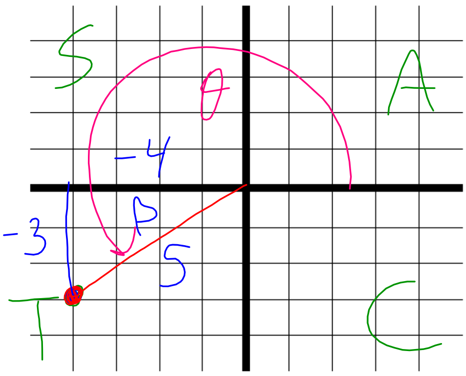
$\therefore \theta = 180 - 53^\circ$

$$\boxed{\theta = 127^\circ}$$

2) Now, choose the point P(-4,-3) on the circumference of the circle.

a) Determine the primary trig raos for the principal angle.

$\sin\beta = \frac{3}{5}$ $\cos\beta = \frac{4}{5}$ $\tan\beta = \frac{3}{4}$



* Q3 (T) only tan is (+)

$\sin\theta = -\sin\beta$ $\cos\theta = -\cos\beta$ $\tan\theta = \tan\beta$

$\boxed{\sin\theta = -\frac{3}{5} \quad \cos\theta = -\frac{4}{5} \quad \tan\theta = \frac{3}{4}}$

b) Determine the principal angle to the nearest degree.

Q3.... $\theta = 180° + \beta$

Find β:

$\sin\beta = \frac{3}{5}$

$\beta = \sin^{-1}\left(\frac{3}{5}\right)$

$\beta = 37°$

$\boxed{\therefore \theta = 217°}$

$(180 + \beta)$

Minds on

Identities

An identity is a mathematical statement that is true for all values of the given variables. If the identity involves fractions, the denominators cannot be zero. Any restrictions on a variable must be stated.

Minds on

Identities You Know

Reciprocal Identities

$$\csc\theta = \frac{1}{\boxed{\sin\theta}} \qquad \sec\theta = \frac{1}{\boxed{\cos\theta}} \qquad \cot\theta = \frac{1}{\boxed{\tan\theta}}$$

We'll accept these as definitions.

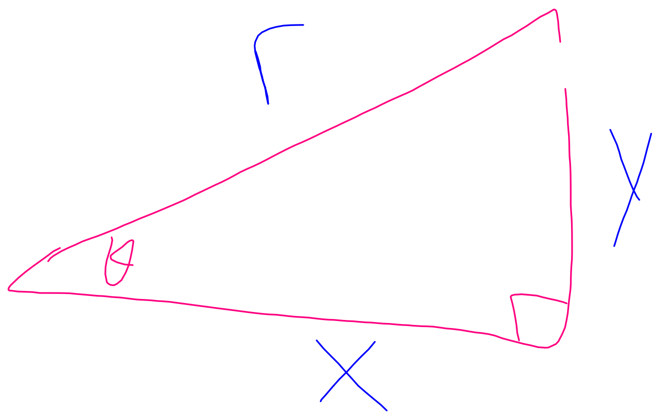
Minds on

Identities You Know

The Basic "Identities"

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

Can be "proven" using sohcahtoa



Action!

Identities You Don't

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Everything in mathematics is built upon a relatively small set of definitions.

Anything that is then introduced must be proven to be accepted.

Prove it!

L.S.

$\tan\theta$

$= \frac{y}{x}$

$\cos\theta \neq 0$

$\theta \neq \cos^{-1} 0$

$\theta \neq 90$

R.S.

$\frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{y}{r}}{\frac{x}{r}}$

$= \frac{y}{\cancel{r}} \times \frac{\cancel{r}}{x}$

$= \frac{y}{x}$

L.S. = R.S.

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$ for all angles θ, where

$0° \leq \theta \leq 360°$ and $\theta \neq 90°, 270°$.

Action!

Identities You Don't

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Your Turn!

L.S.

$= \cot\theta$

$= \frac{1}{\tan\theta}$

$= \frac{1}{\frac{\sin\theta}{\cos\theta}}$

$= 1 \times \frac{\cos\theta}{\sin\theta}$

$= \frac{\cos\theta}{\sin\theta}$

R.S.

$= \frac{\cos\theta}{\sin\theta}$

restrictions

$\sin\theta \neq 0$

$\theta \neq \sin^{-1}(0)$

$\theta \neq 0^\circ$

L.S. = R.S.

$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta}$ where $0^\circ \leq \theta \leq 360^\circ$

and $\theta \neq 0^\circ, 180^\circ, 360^\circ$

Action!

Identities You Don't

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

Prove it!

L.S. R.S.

$= \sin^2\theta + \cos^2\theta$ $= 1$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2 + x^2}{r^2}$

$= \frac{r^2}{r^2}$ by Pythagorean Theorem

$= 1$

L.S. = R.S.

$\therefore \sin^2\theta + \cos^2\theta = 1$ for all θ, $0° \leq \theta \leq 360°$

Action!

Identities You Don't

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

Prove it!

L.S. R.S.

$= 1 + \tan^2\theta$ $= \sec^2\theta$

$= 1 + \dfrac{\sin^2\theta}{\cos^2\theta}$ $= \dfrac{1}{\cos^2\theta}$ * restriction

$= \dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$

Restrictions
$\cos^2\theta \neq 0$
$\cos\theta \neq 0$
$\theta \neq \cos^{-1}(0)$
$\theta \neq 90°, 270°$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$ $\therefore \sin^2\theta + \cos^2\theta = 1$

$= \dfrac{1}{\cos^2\theta}$

$\therefore 1 + \tan^2\theta = \sec^2\theta$ for all θ, $0° \leq \theta \leq 360°$, $\theta \neq 90°, 270°$

Action!

Identities You Don't

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

Your Turn!