What's Going On?

Checking In

Minds on Proving a Pythagorean Identity

Action! Simplifying and Factoring

Consolidation Proving by Factoring

Learning Goal - I will be able to prove trigonometric identities by factoring.

Minds on

Proving a Pythagorean Identity

Pythagorean Identities

$$1+\cot^2\theta=\csc^2\theta$$

LS	RS
$=1+\cot^2\theta$	$=\frac{1}{\sin^2\theta}$
$=1+\frac{\cos^2\theta}{\sin^2\theta}$ by quotient identity	by reciprocal identity
$=\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}$	
$=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}$	
$=\frac{1}{\sin^2\theta}$ by Pythagorean Identity	

$LS=RS$ ∴

$1+\cot^2\theta=\csc\theta$

where $0°\le\theta\le360°$

$\sin^2\theta\neq0$

$\sin\theta\neq0$

$\theta\neq\sin^{-1}0$

$\theta\neq0°,180°,360°$

Action!

Simplifying Trigonometric Expressions

Simplify

a) $1+\cot^2\theta$

$= 1 + \frac{\cos^2\theta}{\sin^2\theta}$ by quotient identity

$= \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta}$ by Pythagorean Identity

$= \csc^2\theta$ by reciprocal identity

b) $\dfrac{\cot\theta}{\cos\theta}$

$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{1}}$ by quotient identity

$= \frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$ by reciprocal identity

c) $(1-\cos\theta)(1+\cos\theta)$

$= 1 + \cos\theta - \cos\theta - \cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta$ by Pythagorean Identity

d) $\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$

$= \dfrac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)}$ diff. of squares

$= \sin\theta + \cos\theta$

Action!

Factoring Trigonometric Expressions

Common Factor

① $\sin\theta\cos\theta + \sin\theta$

$= \sin\theta(\cos\theta + 1)$

② $\tan^2\theta - \sin\theta\tan\theta$

$= \tan\theta(\tan\theta - \sin\theta)$

Difference of Squares

① $\sin^2\theta - \tan^2\theta$

$= (\sin\theta + \tan\theta)(\sin\theta - \tan\theta)$

② $1 - \sin^2\theta$

$= (1 + \sin\theta)(1 - \sin\theta)$

Factor

① $\cos^2\theta + 2\cos\theta - 15$

$x^2 + 2x - 15$

$= (x+5)(x-3)$

$= (\cos\theta + 5)(\cos\theta - 3)$

Consolidation

Proving by Factoring

$$\tan\theta = \frac{\sin\theta + \sin^2\theta}{\cos\theta + \sin\theta\cos\theta}$$

L.S.

$= \tan\theta$

R.S.

$= \dfrac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

by quotient identity