9.1A The Intersection of Lines and Planes

Intersection between a line and a plane can happen three different ways:

Case 1: The line L intersects the plane π at exactly one point, P.

Case 2: The line L does not intersect the plane so it is parallel to the plane.
There are no points of intersection.

Case 3: The line L lies on the plane π. Every point on L intersects the plane.
There are an infinite number of points of intersection.

Example 1: Determine points of intersection between the line L: $\vec{r}$ = (3, 1, 2) + s(1, -4, -8), s ∈ R, and the plane π: 4x + 2y – z – 8 = 0, if any exist.

Example 2: Determine point of intersection between the line L: x = 2 + t, y = 2 + 2t, z = 9 + 8t, t ∈ R , and the plane π: 2x – 5y + z – 6 = 0, if any exist.

Example 3: Determine point of intersection of the line L: $\vec{r} = (3, -2, 1) + s(14, -5, -3)$, $s \in R$, and the plane $x + y + 3z - 4 = 0$, if any exist.

Example 4: Determine points where L: $x = 2 - s$, $y = -1 + 3s$, $z = 4 - 2s$, $s \in R$, and π: $x = -3$ intersect.

9.1B The Intersection of Lines with Lines

Intersection between Two Lines

Intersecting Lines:

Case 1: Intersecting Lines at a Point

Case 2: Coincident Lines

Non-intersecting Lines:

Case 3: Parallel Lines

Case 4: Skew Lines (only exist in R^3)

Example 5: For L_1: $x = -1 + s$, $y = 3 + 4s$, $z = 6 + 5s$, $s \in R$, and L_2: $x = 4 - t$, $y = 17 + 2t$, $z = 30 - 5t$, $t \in R$, determine points of intersection, if any exist.

Example 6: For L_1: $\vec{r} = (-3, 1, 4) + s(-1, 1, 4)$ $s \in R$ and L_2: $\vec{r} = (1, 4, 6) + t(-6, -1, 6)$, $t \in R$, determine points of intersection, if any exist.