

## 9.5 The Distance from a Point to a Line in $\mathbb{R}^2$ and $\mathbb{R}^3$

---

Distance from a Point  $P_0(x_0, y_0)$  to the Line with Equation  $Ax + By + C = 0$

$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$ , where  $d$  represents the distance between point  $P_0(x_0, y_0)$ , and the line defined by  $Ax + By + C = 0$ , where the point does not lie on the line. But we don't really need this since the formula gives the correct value of 0 when the point does lie on the line.

---

**Example 1:** Determine the distance from point  $P(-6, 4)$  to the line  $5x - 3y + 15 = 0$ .

**Example 2:** Determine the distance from point  $P(15, -9)$  to the line  $\vec{r} = (-2, -1) + s(-4, 3)$ ,  $s \in \mathbb{R}$ .

**Example 3:** Calculate the distance between the two parallel lines  $5x - 12y + 60 = 0$  and  $5x - 12y - 60 = 0$

---

**Distance,  $d$ , from a Point,  $P$ , to the line  $\vec{r} = \vec{r}_0 + s\vec{m}, s \in \mathbb{R}$ .**

In  $\mathbb{R}^3$ ,  $d = \frac{|\vec{m} \times \overrightarrow{QP}|}{|\vec{m}|}$ , where  $Q$  is a point on the line and  $P$  is any other point, both of whose coordinates are known, and  $\vec{m}$  is the direction vector of the line.

---

**Example 4:** Determine the distance from point  $P(-1, 1, 6)$  to the line with equation  $\vec{r} = (1, 2, -1) + t(0, 1, 1), t \in \mathbb{R}$ .