What's Going On?

Checking In

Minds on What's the Formula?

Action! Solving Polynomial Equations

Consolidation Solve 'em!

Learning Goal - I will be able to solve polynomial equations by pencil and paper, and on the TI-83 graphing calculator.

Minds on

What's that Formula?

A family is planning on building a silo for grain storage. The silo will consist of a cylindrical base with a semi-sphere on top.

If the total volume is to be **684π** m^3, and the height of the cylindrical portion is **15** m, what are possible values for the radius of the silo?

$V_{cylinder} = \pi r^2 h$

$V_{sphere} = \frac{4\pi r^3}{3}$

$V_{semisphere} = \frac{\frac{4\pi r^3}{3}}{2} = \frac{2}{3}\pi r^3$

$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$684\pi = \pi r^2 (15) + \frac{2}{3}\pi r^3$

Solve for r

$$644\pi = \pi r^2(15) + \frac{2}{3}\pi r^3$$

If we are going to solve this for r, first, we need to set it equal to 0.

$$\frac{2}{3}\pi r^3 + 15\pi r^2 - 644\pi = 0$$

$$\pi\left(\frac{\frac{2}{3}r^3}{\frac{1}{3}} + \frac{15r^2}{\frac{1}{3}} - \frac{644}{\frac{1}{3}}\right) = 0$$

Get rid of the fraction by dividing each term in the brackets by 1/3.

$$\frac{\pi}{3}\left(2r^3 + 45r^2 - 2052\right) = 0$$

find values of r that make this 0. (Guess & check)

Action!

The Factor Theorem Part II

When dealing with a function with a leading coefficient ≠ 1, numbers that can make *f(x) = 0* are of the form ***p/q***, where *p* is a factor of the constant term, and *q* is a factor of the leading coefficient.

$\frac{\pi}{3}(2r^3+45r^2-2052)=0$

Possible values:

Factors of 2052: 1, 2052, 2, 1026, 3, 684, 4, 513, 6, 342, 9, 228

Factors of 2: 1, 2

$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{4}{1}, \pm\frac{4}{2}, \pm\frac{6}{1}, \pm\frac{6}{2}, \pm\frac{9}{1}, \pm\frac{9}{2}$

To the TI-83s!

To the TI-83s!

$\frac{\pi}{3}(2r^3+45r^2-2052)=0$

1. Clear the RAM

2. Enter the function in the brackets above. Hit the Y= button, then enter the function.

3. Hit 2nd Trace (CALC). Choose value.

4. Type a value, hit ENTER. Keep testing values until you get Y = 0.

When $r=6$, $2r^3+45r^2-2052=0$, so $r-6$ is a factor

Divide 6 into $2r^3+45r^2-2052$

$$\begin{array}{r|rrrr} 6 & 2 & 45 & 0 & -2052 \\ & & 12 & 342 & 2052 \\ \hline & 2 & 57 & 342 & 0 \end{array}$$

Rewrite function.

$$\frac{\pi}{3}(r-6)(2r^2+57r+342)$$

solve for r.

Quadratic Formula

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$r = \frac{-57 \pm \sqrt{57^2 - 4(2)(342)}}{2(2)}$$

$$r = \frac{-57 \pm \sqrt{513}}{4}$$

$$r = \frac{-57 + \sqrt{513}}{4} \text{ or } r = \frac{-57 - \sqrt{513}}{4}$$

$r \doteq -6.6$ or $r \doteq -19.9$

negative radius impossible.

$\therefore$ the radius is 6.

Action!

Solving a Cubic

factors 1, 2, 4

factors 1, 3

Solve $4x^3 - 12x^2 - x + 3 = 0$.

Method 1: The factor theorem.

Possible factors to test: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

$x-3$ is a factor!

Divide

$$\begin{array}{r|rrrr} 3 & 4 & -12 & -1 & 3 \\ & & 12 & 0 & -3 \\ \hline & 4 & 0 & -1 & 0 \end{array}$$

$\therefore (x-3)(4x^2-1)=0$

$(x-3)(2x+1)(2x-1)=0$

$\therefore x = 3, -\frac{1}{2}, \frac{1}{2}$

Action!

Solving a Cubic

Solve $4x^3 - 12x^2 - x + 3 = 0$.

Method 2: Factoring by Grouping

$4x^2(x-3) - 1(x-3) = 0$

$(x-3)(4x^2-1) = 0$

$(x-3)(2x+1)(2x-1) = 0$

$\therefore x = 3, -\frac{1}{2}, \frac{1}{2}$

Action!

Solving a Cubic

Solve $4x^3 - 12x^2 - x + 3 = 0$.

Method 3: On the Graphing Calculator

1. Enter the function on the calculator.
2. 2nd Trace (CALC)
3. Choose **2: zero**
4. Provide left and right bounds, and a guess.
5. Repeat steps 3 and 4 for the other zeroes.

$$x = -0.5, 0.5, 3$$

Action!

Where do we meet?

The heights of two orcas above the water in centimetres for a period of 8 seconds can be modelled by the equations:

$h(t) = 20t^3 - 200t^2 + 300t - 200$

$h(t) = 2t^4 - 17t^3 + 27t^2 - 252t + 232$

Determine when the orcas are at the same height over the 8-second period.

Set them equal.

$20t^3 - 200t^2 + 300t - 200 = 2t^4 - 17t^3 + 27t^2 - 252t + 232$

Get everything to one side.

$2t^4 - 37t^3 + 227t^2 - 552t + 432 = 0$

Find a value of t that goes in evenly.

test $\pm 1, \pm 2, \pm 3, \ldots$

$x-4$ is a factor

Divide

$$\begin{array}{r|rrrr} 4 & 2 & -37 & 227 & -552 & 432 \\ & & 8 & -116 & 444 & -432 \\ \hline & 2 & -29 & 111 & -108 & 0 \end{array}$$

$\therefore (x-4)(2x^3-29x^2+111x-108)=0$

find a factor (TI-83)
test $\pm1, \pm2, \pm3, \ldots$

$x-4$ is a factor, again!

$$\begin{array}{r|rrrr} 4 & 2 & -29 & 111 & -108 \\ & & 8 & -84 & 108 \\ \hline & 2 & -21 & 27 & 0 \end{array}$$

$(x-4)(x-4)(2x^2-21x+27)=0$

$(x-4)^2(2x^2-21x+27)=0$

factor

$(x-4)^2(2x^2-3x-18x+27)=0$

$(x-4)^2(x(2x-3)-9(2x-3))=0$

$(x-4)^2(2x-3)(x-9)=0$

$\therefore x=4, \frac{3}{2}, 9$

Pg. 204

2, 6, 7, 8, 16