Learning Goal: I will be able to take derivatives of natural logarithms and natural exponential functions

Minds On: RAFT

Action: Class note + practice

Consolidation:

Minds On

Two New Functions

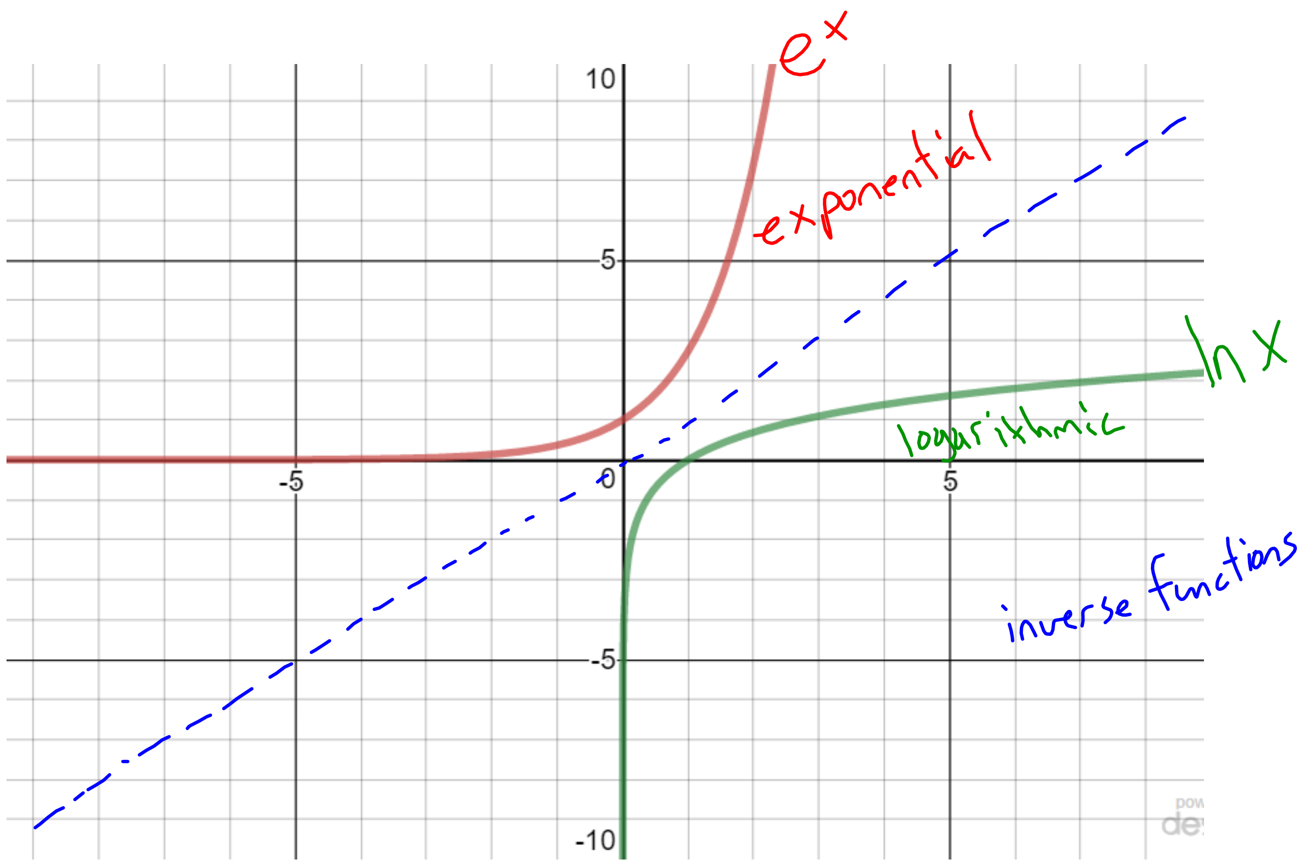
Today we will look at two new functions:

$$f(x) = e^x$$

$$g(x) = \ln x$$

Graph them both in Desmos

What do you notice?


e^x
exponential
$\ln x$
logarithmic
inverse functions
10
5
-5
-10
-5
0
5

Action

Old School Derivative

$e^5 = 148.41$

$$f(x) = e^x$$

x	$f(x)$	EJ	CA	AM	JS	PV	G	$f'(x)$
					$f'(x)$			
-2	0.14	0.44	0.21	0.31	0.18	0.12	0.17	0.24
-1	0.37	0.50	0.40	0.44	0.36	0.34	0.40	0.40
0	1	1.00	0.80	1.00	1	1.00	1.00	0.97
1	2.72	2.67	2.5	2.80	2.44	2.50	2.67	2.60
2	7.39	8.00	7.2	7.20	7.5	7.50	8.00	7.57
3	20.09	18.00	14.00	19.00	20	16.00	20.00	14.5

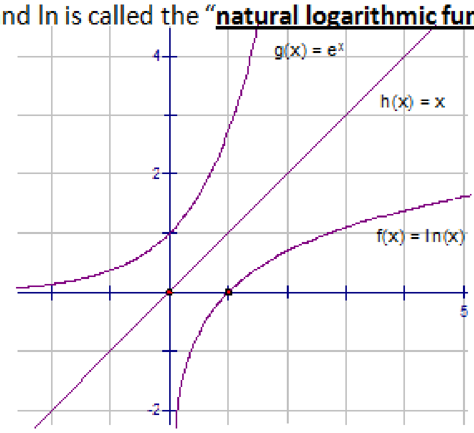
Action

5.1 Derivatives of Exponential Functions

Exponential functions are often used to model rapid change. We are going to look at the function $y = e^x$ and its derivative. The number 'e' is a special irrational number, just like π. It is also referred to as the "**natural number**". Euler's number. 2.718

We learned in Advanced Functions that the inverse of the exponential function is the logarithmic function. For example, the inverse of $y = 2^x$ is $y = \log_2 x$. The inverse of $y = e^x$ would be $y = \log_e x$. This can also be written as $y = \ln x$, and ln is called the "**natural logarithmic function**". "lawn"

Properties of $y = e^x$



$y = e^x$	$y = \ln x$
The domain is {x Є R}	The domain is {x Є R, x > 0}.
The range is {y Є R, y > 0}	The range is {y Є R}
The function passes through (0,1)	The function passes through (1,0)
$e^{\ln x} = x$, $x > 0$	$\ln e^x = x$, $x \in R$
The line y = 0 is the horizontal asymptote	The line x = 0 is the vertical asymptote

Action

Derivative of $f(x) = e^x$

For the function $f(x) = e^x$, $f'(x) = e^x$

Derivative of a Composite Function Involving e^x

In general, if $f(x) = e^{g(x)}$, then $f'(x) = e^{g(x)}g'(x)$ by the chain rule

Action

Example 1: Determine the derivative of $f(x) = e^{3x}$.

$f'(x) = (e^{3x})(3)$

$= 3e^{3x}$

$g(x) = (3x+1)^2$

$= 2(3x+1)^1(3)$

Action

Example 2: Determine the derivative of each function.

a) $g(x) = e^{x^2 - x}$

$g'(x) = (e^{x^2-x})(2x-1)$

$= e^{x^2-x}(2x-1)$

b) $f(x) = x^2 e^x$

$f'(x) = (2x)(e^x) + (x^2)(e^x)$

$= 2xe^x + x^2e^x$

$= e^x(2x+x^2)$

Example 3: Given $f(x) = 3e^{x^2}$, determine f'(-1).

$f'(x) = 3e^{x^2} \cdot 2x$

$= 6xe^{x^2}$

$f'(-1) = 6(-1)e^{(-1)^2}$

$= -6e^1$

$= -6e$

$\boxed{= -16.31}$

Action

Example 4: Determine the equation of the line tangent to $y = \frac{e^x}{x^2}$, where x = 2.

need Slope

1. Find $f'(2)$

$y = e^x x^{-2}$

$\frac{dy}{dx} = (e^x)(x^{-2}) + (e^x)(-2)(x)^{-3}$

$= \frac{x \cdot e^x}{x \cdot x^2} - \frac{2e^x}{x^3}$

$= \frac{xe^x - 2e^x}{x^3}$

$= \frac{e^x(x-2)}{x^3}$

$y = mx + b$

$f'(2) = \frac{e^2(2-2)}{2^3}$

$= 0$

$m = 0$

need a point (x, y)

horizontal line

get a point: $f(2) = \frac{e^2}{(2)^2} = \frac{e^2}{4}$

$\doteq 1.85$

$y = mx + b$

$b = y - mx$

$= 1.85 - 0(2)$

$= 1.85$

$y \doteq 1.85$ or $y = \frac{e^2}{4}$

Consolidation

Application Question

12. The number, N, of bacteria in a culture at time t, in hours, is $N(t) = 1000\left[30 + e^{-\frac{t}{30}}\right]$
    a. What is the initial number of bacteria in the culture?
    b. Determine the rate of change in the number of bacteria at time t.
    c. How fast is the number of bacteria changing when $t = 20$?
    d. Determine the largest number of bacteria in the culture during the interval $0 \le t \le 50$.
    e. What is happening to the number of bacteria in the culture as time passes?

a) $N(0) = 1000\left[30 + e^{-\frac{0}{30}}\right]$
$= 1000\left[30 + e^{0}\right]$
$= 1000(30+1)$
$= 1000(31)$
$= 31{,}000$

b) $N'(t) = 1000\left[0 + e^{-\frac{t}{30}} \times -\frac{1}{30}\right]$
$= 1000\left[-\frac{1}{30} e^{-\frac{t}{30}}\right]$
$= \frac{-1000}{30} e^{-\frac{t}{30}}$
$= -\frac{100}{3} e^{-\frac{t}{30}}$

c) $N'(20) = \frac{-100}{3} e^{-\frac{20}{30}}$
$= -\frac{100}{3} e^{-\frac{2}{3}}$
$\doteq -17$ bacteria/hour

d) $e^{-\frac{t}{30}}$ is always > 0 so $\frac{dN}{dt} \neq 0$ ever (no critical pt.)
$\therefore$ check ends ($t=0$ and $t=50$)
$N(0) = 31{,}000$ max at $t=0$ $\quad N(50) \doteq 30{,}189$

e) Based on answer in part d, the number of bacteria must be constantly decreasing.