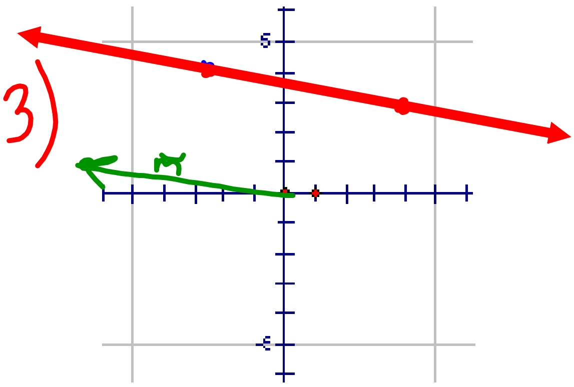
To find out the **vector** and **parametric** equations of a line, we must be given either <u>two distinct points</u> **or** <u>one point and a vector that defines the direction of the line</u>. In either situation, a **direction vector** for the line is necessary.

A **direction vector** is defined to be a nonzero vector $\boxed{\vec{m} = (a, b)}$ parallel (collinear) to the given line. The direction vector $\vec{m} = (a, b)$ is represented by a vector with a tail at the origin and head at the point (a, b). The x and y components of this direction vector are called its **direction numbers**.

Example 1: a) a line passing through P(4, 3) has $\vec{m}$ = (-7, 1) as its direction vector. Sketch the line.

b) A line passes through the points A(½, -3) and B (¾, ½) . Determine the direction vector for this line, and write it using integer components.



b) $\overrightarrow{AB} = \left(\frac{3}{4}-\frac{1}{2}, \frac{1}{2}--3\right)$

$= \left(\frac{1}{4}, \frac{7}{2}\right)$

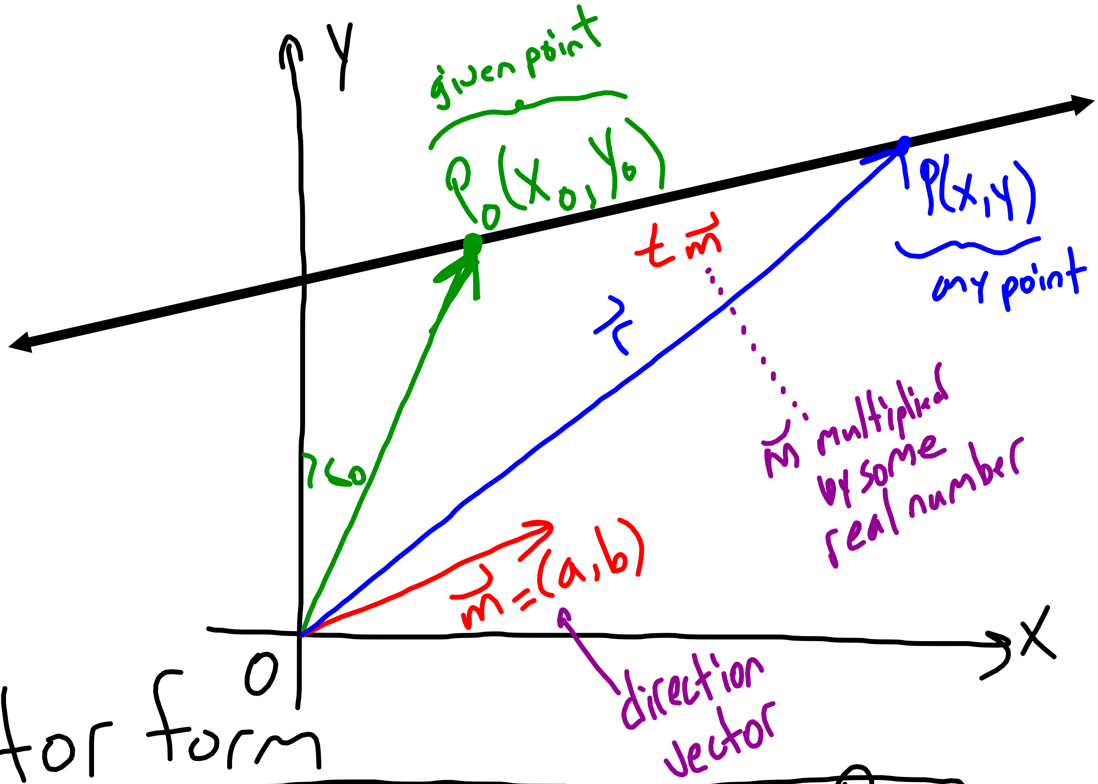
$\overrightarrow{BA} = \left(-\frac{1}{4}, -\frac{7}{2}\right)$

$\cancel{A}$ $\vec{m} = 4\left(\frac{1}{4}, \frac{7}{2}\right) = (1, 14)$

or $\vec{m} = (-1, -14)$

Expressing the Equations of Lines Using Vectors

Read pg. 428 Copy Diagram



vector form

$$\boxed{\vec{r} = \vec{r_0} + t\vec{m}, \; t \in \mathbb{R}}$$

parametric form

$$\boxed{\begin{aligned} x &= x_0 + ta, \; t \in \mathbb{R} \\ y &= y_0 + tb, \; t \in \mathbb{R} \end{aligned}}$$

Vector and Parametric Equations of a Line in R^2

Vector Equation: $\vec{r} = \vec{r_0} + t\vec{m}$, $t \in \mathbf{R}$

Parametric Equations: $x = x_0 + ta$, $y = y_0 + tb$, $t \in \mathbf{R}$

where $\vec{r_0}$ is the vector from (0, 0) to the point (x_0, y_0) and $\vec{m}$ is a direction vector with components (a, b).

Example 2: a) Determine the vector and parametric equations of a line passing through point A(1, 4) with direction vector $\vec{m} = (-3, 3)$. or $\vec{m} = (-1, 1)$ or $\vec{m} = (-2, 2)$

b) Sketch the line, and determine the coordinates of four points on the line.

c) Is either point Q (-21, 23) or point R(-29, 34) on this line?

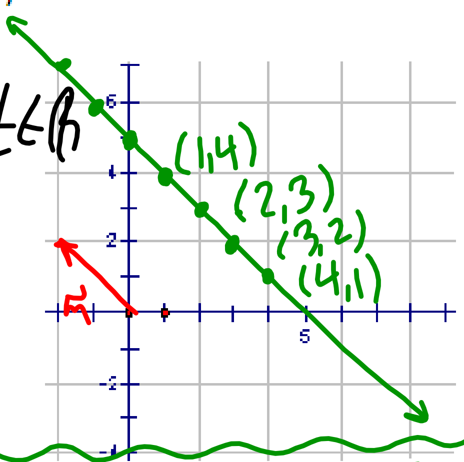
Vector Form

a) $\vec{r} = (1,4) + t(-3,3),\ t \in \mathbb{R}$

$(1,4) + -2(-3,3)$

Parametric Form

$$\left.\begin{array}{l} x = 1 - 3t \\ y = 4 + 3t \end{array}\right\} t \in \mathbb{R}$$



* to get points, set t = different values, when $t = -2$

$(x,y) = (1,4) + -2(-3,3)$

$= (1,4) + (6,-6) = (7,-2)$

c) $Q(-21, 23)$

Using parametric form

$-21 = 1 - 3t$

$3t = 22$

$t = \frac{22}{3}$

$23 = 4 + 3t$

$3t = 19$

$t = \frac{19}{3}$

$\neq$

$\therefore$ $Q(-21,23)$ is not on the line

$R(-29, 34)$

$-29 = 1 - 3t \qquad 34 = 4 + 3t$

$3t = 30 \qquad 3t = 30$

$t = 10 \qquad t = 10$

$=$

$\therefore R(-29, 34)$ is on the line!

* We could also write our vector $\vec{r}$ as

$$\vec{r} = (1, 4) + s(-1, 1)$$

changed $\vec{m}$ from $(3, -3)$ to $(-1, 1)$ or $\frac{1}{3}(3, -3)$

Example 3: a) Find vector and parametric equations for the line containing points E(-1, 5) and F (6, 11).

b) What are the coordinates of the point where this line crosses the x-axis?

c) Can the equation $\vec{r} = (-15, -7) + t(14/3, 4)$, $t \in \mathbf{R}$, also represent the line containing points E and F?

a) Find direction vector

$\overrightarrow{EF} = (6-(-1), 11-5)$

$= (7, 6)$

Vector Form

$\vec{r} = (-1, 5) + t(7, 6), \quad t \in \mathbb{R}$

$$\left.\begin{aligned} x &= -1 + 7t \\ y &= 5 + 6t \end{aligned}\right\} t \in \mathbb{R}$$

b) When the line crosses the x-axis $y=0$

use parametric form with $y=0$

$x=-1+7t, \quad y=5+6t$

$0=5+6t$

$t=\frac{-5}{6}$

$x=-1+7\left(\frac{-5}{6}\right)$

$x=-1-\frac{35}{6}$

$x=\frac{-6}{6}-\frac{35}{6}$

$x=\frac{-41}{6}$

$\therefore$ the line crosses the x-axis at $\left(\frac{-41}{6}, 0\right)$

$\vec{r} = (-1, 5) + t(7, 6)$

$\vec{r} = (-15, -7) + t(\frac{14}{3}, 4)$?

equivalent?

can we write $k(7,6)$ and get $(\frac{14}{3}, 4)$?

$6 \cdot \frac{2}{3} = 4$ ✓

$7 \cdot \frac{2}{3} = \frac{14}{3}$

$\therefore$ the lines are collinear

Do they share a point?

is $(-15,-7)$ on our original line?

$$-15 \rightarrow x = -1 + 7t$$

$$-7 \rightarrow y = 5 + 6t$$

$$-15 = -1 + 7t$$
$$7t = -14$$
$$t = -2$$

✓

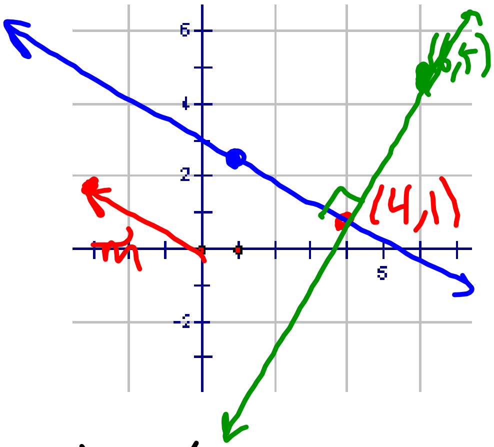
$$-7 = 5 + 6t$$
$$6t = -12$$
$$t = -2$$

$\therefore$ the vector form equation

$\vec{r} = (-15, -7) + t\left(\frac{14}{3}, 4\right)$ can

also represent the line through E + F.

Example 4: Determine a vector equation for the line that is perpendicular to $\vec{r} = (4, 1) + s(-3, 2)$, $x \in \mathbf{R}$, and passes through point P(6, 5).



Find green line!

Find direction vector $\vec{n} = (a, b)$

Dot product of direction vectors = 0

$$\vec{m} \cdot \vec{n} = 0$$

$$(-3, 2) \cdot (a, b) = 0$$

$$-3a + 2b = 0$$

$$b = \frac{3}{2}a$$

Let $a = 2$

then $b = 3$ $\therefore$ direction vector $= \vec{n} = (2, 3)$

$\therefore$ our vector is $\vec{r} = (6, 5) + t(2, 3)$, $t \in R$