

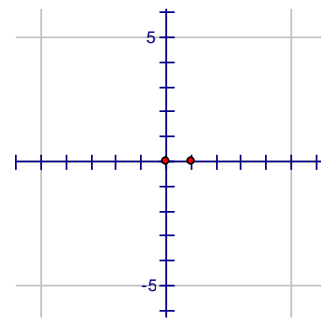
## 8.1 Vector and Parametric Equations of a Line in $\mathbb{R}^2$

To find out the **vector** and **parametric** equations of a line, we must be given either two distinct points or one point and a vector that defines the direction of the line. In either situation, a **direction vector** for the line is necessary.

A **direction vector** is defined to be a nonzero vector  $\vec{m} = (a, b)$  parallel (collinear) to the given line. The direction vector  $\vec{m} = (a, b)$  is represented by a vector with a tail at the origin and head at the point  $(a, b)$ . The x and y components of this direction vector are called its **direction numbers**.

**Example 1:** a) a line passing through  $P(4, 3)$  has  $\vec{m} = (-7, 1)$  as its direction vector. Sketch the line.

b) A line passes through the points  $A(\frac{1}{2}, -3)$  and  $B(\frac{3}{4}, \frac{1}{2})$ . Determine the direction vector for this line, and write it using integer components.



**Expressing the Equations of Lines Using Vectors**

---

### Vector and Parametric Equations of a Line in $\mathbf{R}^2$

Vector Equation:  $\vec{r} = \vec{r}_0 + t\vec{m}$ ,  $t \in \mathbf{R}$

Parametric Equations:  $x = x_0 + ta$ ,  $y = y_0 + tb$ ,  $t \in \mathbf{R}$

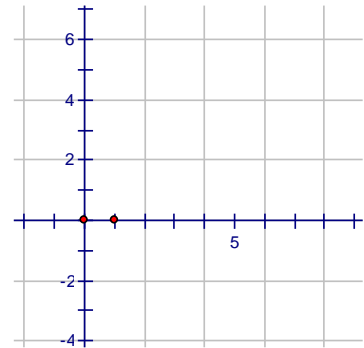
where  $\vec{r}_0$  is the vector from  $(0, 0)$  to the point  $(x_0, y_0)$  and  $\vec{m}$  is a direction vector with components  $(a, b)$ .

---

**Example 2:** a) Determine the vector and parametric equations of a line passing through point  $A(1, 4)$  with direction vector  $\vec{m} = (-3, 3)$ .

b) Sketch the line, and determine the coordinates of four points on the line.

c) Is either point  $Q(-21, 23)$  or point  $R(-29, 34)$  on this line?



- Example 3:** a) Find vector and parametric equations for the line containing points E(-1, 5) and F (6, 11).  
b) What are the coordinates of the point where this line crosses the x-axis?  
c) Can the equation  $\vec{r} = (-15, -7) + t(14/3, 4)$ ,  $t \in \mathbf{R}$ , also represent the line containing points E and F?

**Example 4:** Determine a vector equation for the line that is perpendicular to  $\vec{r} = (4, 1) + s(-3, 2)$ ,  $s \in \mathbf{R}$ , and passes through point P(6, 5).

