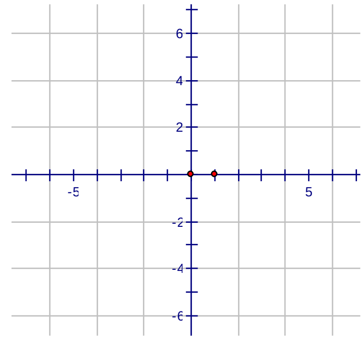
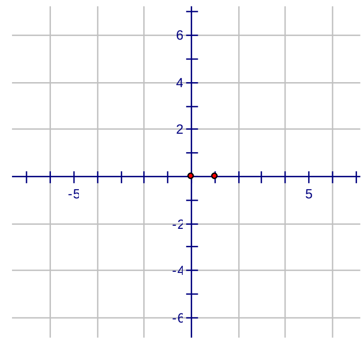
8.2 Cartesian Equation of a Line

Direction Vectors and Slope



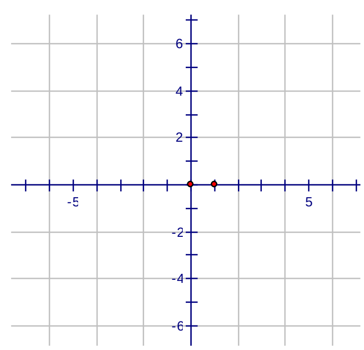
In the diagram, a line segment AB with slope m =   is shown with a run of   and a rise of   . The vector   is used to describe the direction of this line or any line parallel to it, with no restriction on the direction numbers a and b. In practice, a and b can be any two real numbers when describing a direction vector. If the direction vector of a line is m = (a, b), this corresponds to a slope of m = b/a except when a = 0. (why?   )

Example 1: Determine the equivalent vector and parametric equations of the line y = (3/4)x + 2.



Example 2: For the line with equation $\vec{r}$ = (3, -6) + s(-1, -4), s ∈ R, determine the equivalent slope-y-intercept form.

Example 3: Determine the Cartesian form of the line with the equation $\vec{r}$ = (1, 4) + s(0, 2), s ∈ R.

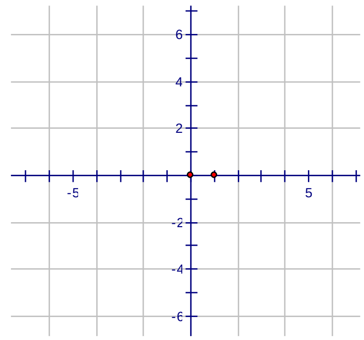


Cartesian Equation of a Line in R^2

In R^2, the Cartesian equation of a line (or scalar equation) is given by $Ax + By + C = 0$, where a normal to this line is $\vec{n} = (A, B)$.

A **normal** to this line is a vector drawn from the origin perpendicular to the given line to the point N(A, B).

Example 4: Determine the Cartesian equation of the line passing through A(4, -2), which has $\vec{n} = (5, 3)$ as its normal.

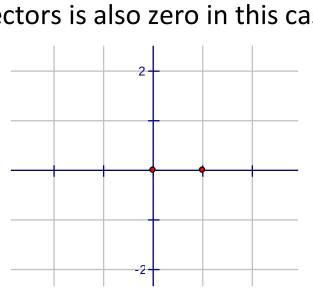


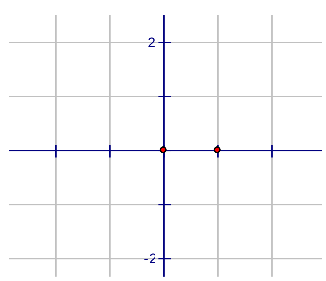
Parallel and Perpendicular Lines and their Normals

If the lines L_1 and L_2 have normals $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$, respectively, we know the following:

1. The two lines are parallel IFF their normals are scalar multiples, $\overrightarrow{n_1} = k\overrightarrow{n_2}$, $k \in R$, $k \neq 0$. It follows that the direction vectors of the lines are also scalar multiples in this case.

2. The two lines are perpendicular IFF their dot product is zero, $\overrightarrow{n_1} \bullet \overrightarrow{n_2} = 0$. It follows that the dot product of the direction vectors is also zero in this case.





Example 5: a) Show that the lines L_1: $3x - 4y - 6 = 0$ and L_2: $6x - 8y + 12 = 0$ are parallel and non-coincident.

b) For what value of k are the lines L_3: $kx + 4y - 4 = 0$ and L_4: $3x - 2y - 3 = 0$ perpendicular lines?

Example 6: Determine the acute angle formed at the point of intersection created by the following pair of lines:

L_1: $(x,y) = (2,2) + s(-1,3),\ s \in R$ L_2: $(x,y) = (5,1) + t(3,4),\ t \in R$