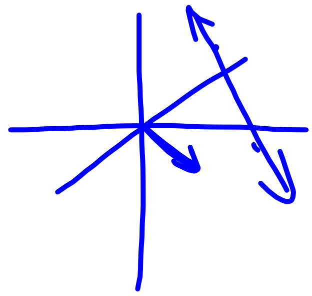
Goal: Work with Vector, Parametric, and Symmetric Equations of a Line in R^3

Agenda

1. Minds On: Direction vectors in 3 space
2. Action: Note on 8.3
3. Consolidation: Exit Question
   Page 449 #5cf

Minds On

A line passes through the points A (-1, 3, 5) and B(-3, 3, -4). Calculate possible direction vectors for this line.



$\vec{m} = \overrightarrow{AB}$ or $\overrightarrow{BA}$

$= (-3-(-1),\ 3-3,\ -4-5)$

$= (-2, 0, -9)$

$= (2, 0, 9)$ $\rightarrow$ $\boxed{\therefore \text{ any direction vector } t(2,0,9),\ \{t \in \mathbb{R} \mid t \neq 0\}}$

$= (1, 0, 4.5)$

Vector and Parametric Equations of Lines in R^3

Vector Equation: $\vec{r} = \overrightarrow{r_o} + t\overrightarrow{m}, t \epsilon R$

Parametric Equations: $x = x_0 + ta$, $y = y_0 + tb$, $z = z_0 + tc$, $t \in R$, where $\overrightarrow{r_o} = (x_0, y_0, z_0)$, the vector from the origin to a point on the line and $\overrightarrow{m} = (a, b, c)$ is a direction vector of the line.

Example 2: Determine the vector and parametric equations of the line passing through P(-2, 3, 5) and Q(-2, 4, -1).

$\vec{r} = \vec{r}_0 + t\vec{m}$

$\vec{m} = \overrightarrow{PQ}$ or $\overrightarrow{QP}$

$= (0, 1, -6)$ $= (0, -1, 6)$

$t \in \mathbb{R}, t \neq 0$

$\vec{r}_0 = P(-2, 3, 5)$

$\therefore$ the vector eq'n is $\vec{r} = (-2, 3, 5) + t(0, 1, -6)$

$\therefore$ the param eq'n is $\begin{cases} x = -2 \\ y = 3 + t \\ z = 5 - 6t \end{cases}$

Example 3: a) Show that the following are vector equations for the same line: L_1: $\vec{r}$ = (-1, 0, 4) + s(-1, 2, 5), s ∈ R, and L_2: $\vec{r}$ = (4, -10, -21) + m(-2, 4, 10), m ∈ R

$\vec{m}_1 = (-1, 2, 5)$

$\vec{m}_2 = (-2, 4, 10)$

$\vec{m}_2 = 2\vec{m}_1 \quad \therefore L_1 \parallel L_2$

If they are the same, they must share all points.

check if (-1,0,4) is on L_2.

param. eq'ns from L_2:

	$x = 4 - 2m$	$y = -10 + 4m$	$z = -21 + 10m$
sub. in pt.	$-1 = 4 - 2m$	$0 = -10 + 4m$	$4 = -21 + 10m$
	$\frac{-5}{-2} = m$	$\frac{10}{4} = m$	$\frac{25}{10} = m$
	$\frac{5}{2} = m$	$\frac{5}{2} = m$	$\frac{5}{2} = m$

✓ all the same!

$\boxed{\therefore L_1 = L_2 .}$

b) Show that the following are vector equations for different lines: L_3: $\vec{r}$ = (1, 6, 1) + p(-1, 1, 2), p ∈ R, and
L_4: $\vec{r}$ = (-3, 10, 12) + k(1/2, -(1/2), -1), k ∈ R

$\vec{m}_3 = -2\vec{m}_4 \quad \therefore L_3 \parallel L_4$

$\left.\begin{array}{l} 8 = k \\ 8 = k \\ 11 = k \end{array}\right\} \times \qquad \left.\begin{array}{l} 4 = p \\ 4 = p \\ \frac{11}{2} = p \end{array}\right.$

$\therefore L_3 \neq L_4$

Symmetric Equations of a Line in R^3

$\vec{r} = \vec{r}_0 + t\vec{m}$
$= (x_0, y_0, z_0) + t(a, b, c)$

The symmetric equation of a line is derived from using its parametric equations and solving for the parameter in each component, as follows:

$x = x_0 + ta \Rightarrow t = \frac{x - x_0}{a}$

$y = y_0 + tb \Rightarrow t = \frac{y - y_0}{b}$

$z = z_0 + tc \Rightarrow t = \frac{z - z_0}{c}$

$\therefore$ Symm eq'n

$$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

Example 4: Write the symmetric equations of the line passing through each pair of points:

a) A(-1, 5, 7) and B(3, -4, 8) b) P(-2, 3, 1) and Q(4, 3, -5)

c) X(-1, 2, 5) and Y(-1, 3, 9)

a) $\vec{m} = \overrightarrow{AB}$

$= (a, b, c)$

$= (4, -9, 1)$

$$\frac{x+1}{4} = \frac{y-5}{-9} = \frac{z-7}{1}$$

$$\frac{x+1}{4} = \frac{y-5}{-9} = z-7$$

b) $\vec{m} = \overrightarrow{PQ}$

$= (6, 0, -6)$

$$\frac{x+2}{6} = \frac{y-3}{0} = \frac{z-1}{-6}$$

vertical Line ☹

$$\frac{x+2}{6} = \frac{z-1}{-6}, \quad y = 3$$

c) $\vec{m} = \overrightarrow{XY}$

$= (0, 1, 4)$

$$x = -1, \quad y-2 = \frac{z-5}{4}$$

Exit Ques:

p. 449 # 5cf