8.3 Vector, Parametric, and Symmetric Equations of a Line in R^3

MINDS ON: A line passes through the points A (-1, 3, 5) and B(-3, 3, -4). Calculate possible direction vectors for this line.

Vector and Parametric Equations of Lines in R^3

Vector Equation: $\vec{r} = \vec{r_o} + t\vec{m}, t \epsilon R$

Parametric Equations: $x = x_0 + ta$, $y = y_0 + tb$, $z = z_0 + tc$, $t \in R$, where $\vec{r_o} = (x_0, y_0, z_0)$, the vector from the origin to a point on the line and $\vec{m} = (a, b, c)$ is a direction vector of the line.

Example 1: Determine the vector and parametric equations of the line passing through P(-2, 3, 5) and Q(-2, 4, -1).

Example 2: a) Show that the following are vector equations for the same line:
L_1: $\vec{r}$ = (-1, 0, 4) + s(-1, 2, 5), s ∈ R, and L_2: $\vec{r}$ = (4, -10, -21) + m(-2, 4, 10), m ∈ R
b) Show that the following are vector equations for different lines:
L_3: $\vec{r}$ = (1, 6, 1) + p(-1, 1, 2), p ∈ R, and L_4: $\vec{r}$ = (-3, 10, 12) + k(1/2, -(1/2), -1), k ∈ R

Symmetric Equations of a Line in R^3

The symmetric equation of a line is derived from using its parametric equations and solving for the parameter in each component, as follows:

Example 3: Write the symmetric equations of the line passing through each pair of points:

a) A(-1, 5, 7) and B(3, -4, 8)

b) P(-2, 3, 1) and Q(4, 3, -5)

c) X(-1, 2, 5) and Y(-1, 3, 9)