Learning Goal: I will be able to write equations of a plane in vector and parametric form.

Minds On: Slope of a line formula

Action: Note and practice

Consolidation: Choose your Exit Question

Minds On

Think/Pair/Share

1. What different sets of given information can lead you to write the equation of a line (in any form)?

slope + point
two points
two intercepts
point + intercept
same

2. What do you think you would need to define a plane ?

- 3 points
- line + point
- 2 intersecting lines
- 2 // and non-coincident lines

Action

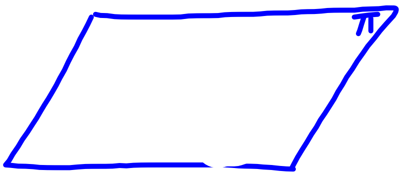
8.4 Vector and Parametric Equations of a Plane

Planes are flat surfaces that extend infinitely far in all directions. To represent planes, parallelograms are used to represent a small part of the plane, which we call π. In real life, we can use a wall, a piece of paper, or a hockey ice surface to represent a plane.

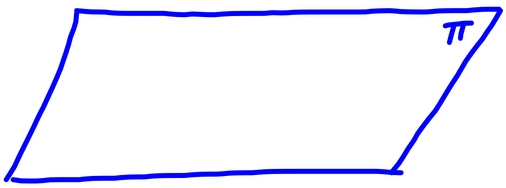
Action

A plane can be determined if we are given any of the following four sets of information:

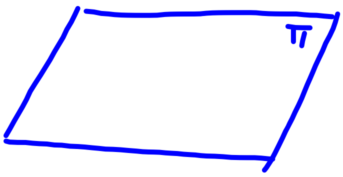
a) a line and a point not on the line



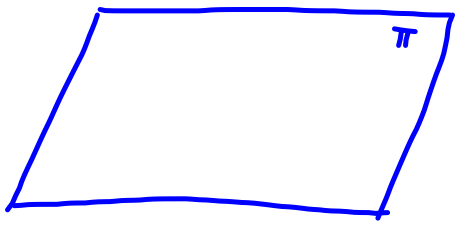
b) three noncollinear points (3 points not on a line)



c) two intersecting lines



d) two parallel and non-coincident lines



Action

Vector and Parametric Equations of a Plane in R³

In R^3, a plane is determined by a vector $\overrightarrow{r_o} = (x_0, y_0, z_0)$ where (x_0, y_0, z_0) is a point on the plane, and two noncollinear vectors $\vec{a}$= (a_1, a_2, a_3) and $\vec{b}$ = (b_1, b_2, b_3).

Vector Equation of a Plane: $\vec{r} = \overrightarrow{r_0} + s\vec{a} + t\vec{b}, s, t \epsilon R$ or equivalently $(x, y, z) = (x_0, y_0, z_0) + s(a_1, a_2, a_3) + t(b_1, b_2, b_3)$

Parametric Equations of a Plane: $x = x_0 + sa_1 + tb_1$,

$y = y_0 + sa_2 + tb_2$, $z = z_0 + sa_3 + tb_3$, $s, t \in R$.

The vectors $\vec{a}$ and $\vec{b}$ are the direction vectors for the plane. When determining the equation of a plane, we need two direction vectors. Any pair of noncollinear vectors are coplanar, so they can be used as direction vectors for a plane.

Action

Example 1: a) Determine a vector equation and the corresponding parametric equations for the plane that contains the points A(-1, 3, 8), B(-1, 1, 0), and C(4, 1, 1).

b) Do either of the points P(14, 1, 3) or Q(14, 1, 5) lie on this plane?

$\vec{r} = \vec{r}_0 + s\vec{a} + t\vec{b}$

$\vec{r} = (x_0, y_0, z_0) + s(a_1, a_2, a_3) + t(b_1, b_2, b_3)$

Need

- 2 direction vectors $\vec{m}_1$ and $\vec{m}_2$
- ✓ 1 point

$\vec{AB} = (0, -2, -8)$

$\vec{AC} = (5, -2, -7)$

$\vec{r} = (-1, 3, 8) + s(0, -2, -8) + t(5, -2, -7)$

$x = -1 + 5t$

$y = 3 - 2s - 2t$

$z = 8 - 8s - 7t$

b) $P(14, 1, 3)$

Use parametric equations

$(14) = -1 + 5t$

$\boxed{t = 3}$

$1 = 3 - 2s - 2t$

$1 = 3 - 2s - 2(3)$

$1 = 3 - 2s - 6$

$4 = -2s$

$\boxed{s = -2}$

$z = 8 - 8s - 7t$

L.S. R.S.

$= (3)$ $= 8 - 8(-2) - 7(3)$

$= 3$ $= 8 + 16 - 21$

$= 24 - 21$

$= 3$

$\therefore$ L.S. = R.S., point $P(14, 1, 3)$ is on the plane $\vec{r}$.

$Q(14, 1, 5)$

$x = -1 + 5t$

$y = 3 - 2s - 2t$

$z = 6 - 6s - 7t$

$(14) = -1 + 5t$

$\boxed{t = 3}$

$(1) = 3 - 2s - 2(3)$

$1 = 3 - 2s - 6$

$\boxed{s = -2}$

LS	RS
(5)	$= 8 - 8(-2) - 7(3)$
5	$= 8 + 16 - 21$
5	$= 24 - 21$
5	$= 3$

L.S. $\neq$ R.S., $\therefore$ (14,1,5) ain't on plane $\vec{r}$.

Action

Example 2: A plane π has $\vec{r} = (6, -2, -3) + s(1, 3, 0) + t(2, 2, -1)$, $s, t \in \mathbb{R}$, as its equation. Determine the point of intersection between π and the z-axis.

When intersecting z-axis, $x = y = 0$.

$\therefore (x, y, z) = (0, 0, z)$

Parametric Form

$x = 6 + s + 2t$ ①
$y = -2 + 3s + 2t$ ②
$z = -3 - t$ ③

1. sub $x = 0$ into ①

$0 = 6 + s + 2t$

$\boxed{s = -6 - 2t}$ ④

2. sub $y = 0$ and ④ into ②

$0 = -2 + 3(-6 - 2t) + 2t$

$0 = -2 - 18 - 6t + 2t$

$20 = -4t$

$\boxed{t = -5}$ ⑤

3. Sub ⑤ into ③, solve for z

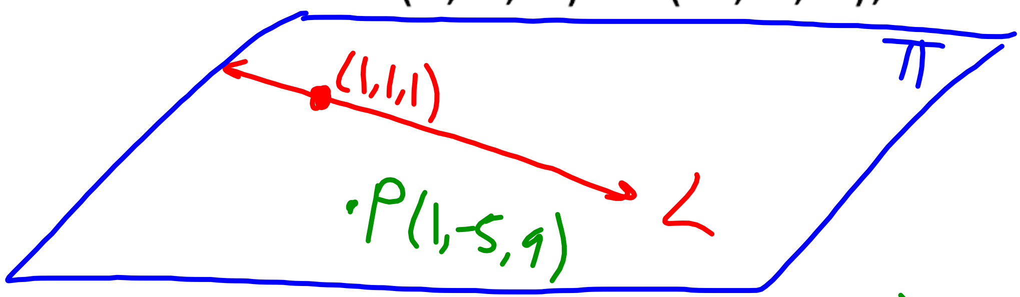
$z = -3 - (-5)$

$z = 2$

$\therefore$ the plane crosses the z-axis @ the point $(0, 0, 2)$

Action

Example 3: Determine the vector and parametric equations of the plane containing the point P(1, -5, 9) and the line L: $\vec{r}$ = (1, $\overset{r_0}{1}$, 1) + s(-1, 1, 0), s ∈ R.



Find $\vec{m}_2 = \overrightarrow{Pr_0} = (1,1,1) - (1,-5,9)$

$= (1-1, 1+5, 1-9)$

$\vec{m}_2 = (0,6,-8) = 2(0,3,-4)$

$\vec{m}_1 = (-1,1,0)$

$\vec{r} = (1,1,1) + s(-1,1,0) + t(0,3,-4), \ s,t \in \mathbb{R}$

$$\left.\begin{aligned} x &= 1 - s \\ y &= 1 + s + 3t \\ z &= 1 - 4t \end{aligned}\right\} s,t \in \mathbb{R}$$

Consolidation

Exit Question
CHOOSE ONE

1. Determine the equation of the plane that contains the point A(-2, 2, 3) and the line $\vec{r} = m(1, -1, 7)$

OR

2. Determine two pairs of direction vectors that can be used to represent the xy-plane in three-space.