8.4 Vector and Parametric Equations of a Plane

Planes are flat surfaces that extend infinitely far in all directions. To represent planes, parallelograms are used to represent a small part of the plane, which we call π. In real life, we can use a wall, a piece of paper, or a hockey ice surface to represent a plane.

A plane can be determined if we are given any of the following four sets of information:

a) a line and a point not on the line

b) three noncollinear points (3 points not on a line)

c) two intersecting lines

d) two parallel and non-coincident lines

Vector and Parametric Equations of a Plane in R^3

In R^3, a plane is determined by a vector $\vec{r_o} = (x_0, y_0, z_0)$ where (x_0, y_0, z_0) is a point on the plane, and two noncollinear vectors $\vec{a}$= (a_1, a_2, a_3) and $\vec{b}$ = (b_1, b_2, b_3).

Vector Equation of a Plane: $\vec{r} = \vec{r_0} + s\vec{a} + t\vec{b}, s, t \epsilon R$ or equivalently
$(x, y, z) = (x_0, y_0, z_0) + s(a_1, a_2, a_3) + t(b_1, b_2, b_3)$

Parametric Equations of a Plane: $x = x_0 + sa_1 + tb_1$, $y = y_0 + sa_2 + tb_2$, $z = z_0 + sa_3 + tb_3$, $s, t \in R$.

The vectors $\vec{a}$ and $\vec{b}$ are the direction vectors for the plane. When determining the equation of a plane, we need two direction vectors. Any pair of noncollinear vectors are coplanar, so they can be used as direction vectors for a plane.

Example 1: a) Determine a vector equation and the corresponding parametric equations for the plane that contains the points A(-1, 3, 8), B(-1, 1, 0), and C(4, 1, 1).

b) Do either of the points P(14, 1, 3) or Q(14, 1, 5) lie on this plane?

Example 2: A plane π has $\vec{r}$ = (6, -2, -3) + s(1, 3, 0) + t(2, 2, -1), s, t ∈ R, as its equation. Determine the point of intersection between π and the z-axis.

Example 3: Determine the vector and parametric equations of the plane containing the point P(1, -5, 9) and the line L: $\vec{r}$ = (1, 1, 1) + s(-1, 1, 0), s ∈ R.