

8.5 The Cartesian Equation of a Plane

Minds On

So far, we've seen all the following types of equations for lines and only some of them for planes. Fill in the table with examples of each type we have already discussed in class. Work with a partner to discuss possible examples and to make predictions about any we've not yet covered.

| Equation | LINES | PLANES |
|----------------------|-------|--------|
| Vector | | |
| Parametric | | |
| Symmetric | | |
| Cartesian/ Scalar | | |

Picture it!

Using a 3D model (such as your textbook) and a plane (such as this piece of paper), you can picture a line L that is perpendicular to the plane, π . For any plane in \mathbb{R}^3 , there is only one possible line that can be drawn through the origin perpendicular to the plane. This line is called the **normal axis** for the plane, and the direction vector is called the **normal** to the plane. The direction of the normal axis is given by a vector joining the origin to any point on the normal axis. This means there are an infinite number of normals for any plane.

Cartesian Equation of a Plane

The Cartesian (or scalar) equation of a plane in \mathbb{R}^3 is of the form $Ax + By + Cz + D = 0$, with normal $\vec{n} = (A, B, C)$. The normal \vec{n} is a nonzero vector perpendicular to all vectors in the plane.

Try on your own or with a partner: The point $A(1, 2, 2)$ is a point on the plane with normal $\vec{n} = (-1, 2, 6)$. Determine the Cartesian equation of this plane.

Example 1: Determine the Cartesian equation of the plane containing the points $A(-1, 2, 5)$, $B(3, 2, 4)$, and $C(-2, -3, 6)$.

Example 2: Determine the vector and parametric equations of the plane with Cartesian equation $x - 2y + 5z - 6 = 0$. Perform a check to verify your answer.

Parallel and Perpendicular Planes – Picture it!

1. If π_1 and π_2 are two perpendicular planes, with normals \vec{n}_1 and \vec{n}_2 , respectively, their normals are perpendicular (that is, the dot product is zero).
 2. If π_1 and π_2 are two parallel planes, with normals \vec{n}_1 and \vec{n}_2 , respectively, their normals are parallel (that is $\vec{n}_1 = k\vec{n}_2$) for all nonzero real numbers k.
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Angle between Intersecting Planes

The angle, θ , between two planes, π_1 and π_2 , with normals of \vec{n}_1 and \vec{n}_2 , respectively, can be calculated using the formula $\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$.

Example 3: Determine the acute and obtuse angles between the two planes

$$\pi_1: x - y - 2z + 3 = 0 \quad \text{and} \quad \pi_2: 2x + y - z + 2 = 0.$$

Exit Question

a) Show that the planes $\pi_1: 2x - 3y + z - 1 = 0$ and $\pi_2: 4x - 3y - 17z = 0$ are perpendicular.

b) Show that the planes $\pi_3: 2x - 3y + 2z - 1 = 0$ and $\pi_4: 2x - 3y + 2z - 3 = 0$ are parallel but not coincident.