Learning Goal: I will be able to determine slopes of tangents using the difference quotient.

Minds On: Secant vs. Tangent

Action: Investigations 1 & 2 on page 12
Note and examples

Consolidation: Practice p.18
Exit Question

Minds On

Warm-up Question

Please start this as you come in so we can get to today's lesson.

Simplify by rationalizing the denominator.

$$\frac{2\sqrt{3}+3\sqrt{10}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{2\sqrt{18}+2\sqrt{15}+3\sqrt{60}+3\sqrt{50}}{6-5}$$

$$= 2\sqrt{9\times 2}+2\sqrt{15}+3\sqrt{4\times 15}+3\sqrt{2\times 25}$$

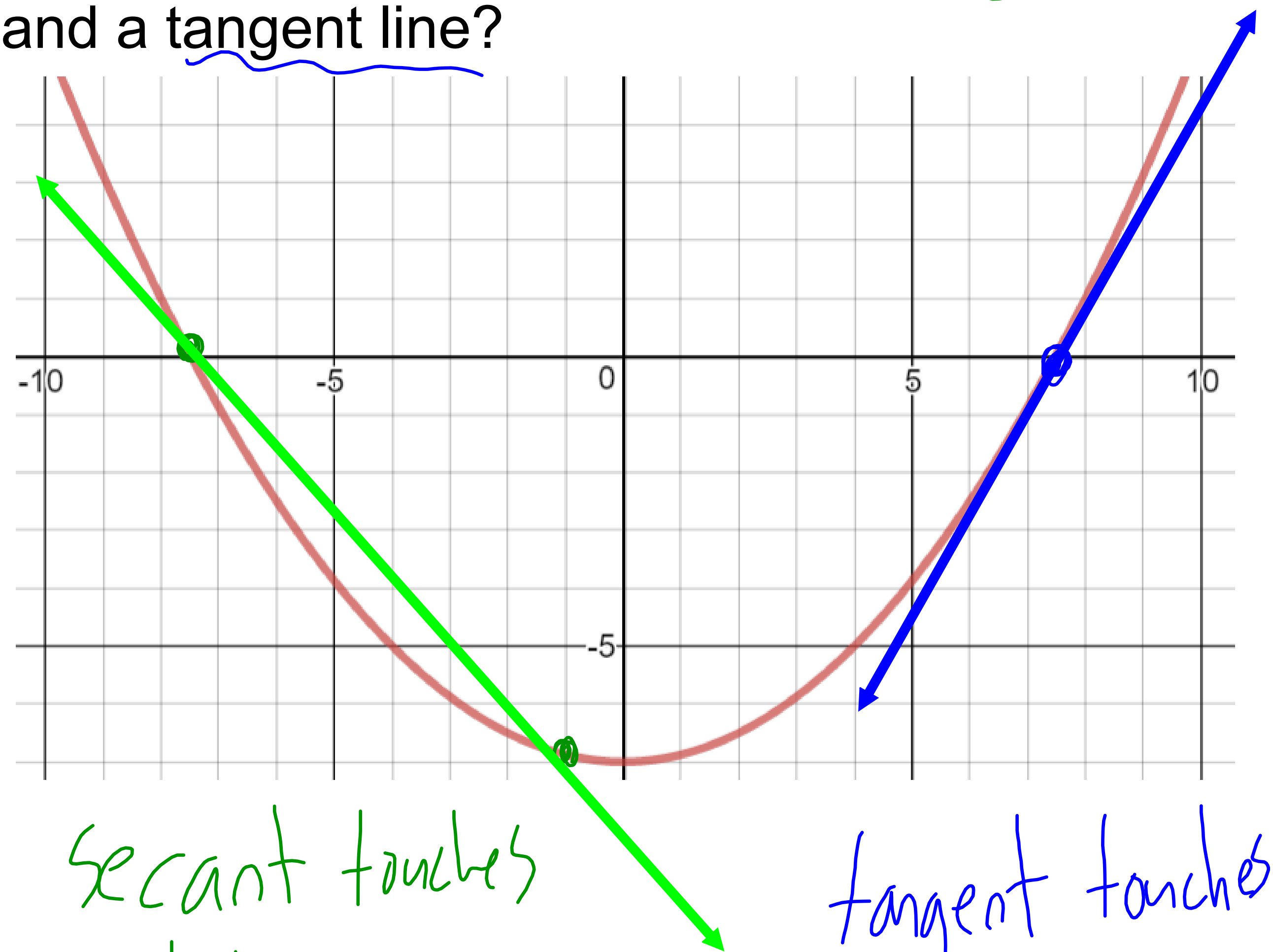
$$= 6\sqrt{2}+2\sqrt{15}+6\sqrt{15}+15\sqrt{2}$$

$$= 21\sqrt{2}+8\sqrt{15}$$

Minds On

Secants vs. Tangents

What is the difference between a secant line and a tangent line?



secant touches twice

tangent touches once

Investigation 1:

a) Determine the y-coordinates of the following points that lie on the graph of the parabola $y = x^2$:

i) Q1 (3.5,y)	ii) Q2 (3.1,y)	iii) Q3 (3.01,y)	iv) Q4 (3.001,y)

b) Calculate the slopes of the secants through P(3,9) and the points Q1, Q2, Q3 and Q4.

i) 6.5	ii) 6.1	iii) 6.01	iv) 6

c) Based on your results from part B, estimate the slope of the tangent at the point (3,9).

6

You just found the slope of the tangent by finding what is called the ***limiting value*** of the slopes of a sequence of secants. To get Q values as close as possible to our point P

$3+h, f(3+h)$

on the parabola $y = x^2$, we write the point Q as being $(3 + h, (3 + h)^2)$, where h is a really small non-zero number.

The value of h determines the position of Q on the parabola. As the point Q slides closer and closer to P, h will become smaller and smaller and get closer to zero. In this case, we say that **"h approaches zero"** which is written as **"h → 0".**

Investigation 2:

a) Determine an expression for the slope of the secant PQ through points $\boxed{P(3,9)}$ and $\boxed{Q(3 + h, (3 + h)^2)}$.

$x_1 \quad y_1$

$x_2 \quad y_2$

$$m = \frac{f(a+h) - f(a)}{h}$$

$$m = \frac{(3+h)^2 - 9}{(3+h) - 3}$$

$$m = \frac{\cancel{9} + 6h + h^2 - \cancel{9}}{\cancel{3} + h - \cancel{3}}$$

$$m = \frac{h^2 + 6h}{h}$$

$$m = \frac{\cancel{h}(h+6)}{\cancel{h}}$$

$$\boxed{m = h + 6}$$

b) Explain how you could use the expression in part A to predict the slope of the tangent to the parabola $f(x) = x^2$ at point P(3,9).

$m = 6 + h$

because h is getting closer to 0, m is getting closer to 6+0 or just 6!

Action

In general:

Let P(a, f(a)) be a fixed point on the graph of y = f(x), and let Q(x,y) = Q(x,f(x)) represent any other point on the graph. If Q is a horizontal distance of h units from P, then x = (a + h) and y = f(a + h). Point Q then has coordinates Q(a + h, f(a + h)).

The slope of the secant PQ is $\frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{(a+h)-a} = \boxed{\frac{f(a+h) - f(a)}{h}}$

This quotient is fundamental to calculus and is referred to as the **difference quotient**. Therefore, the slope ***m*** of the tangent at P(a, f(a)) is the limit of the slope of the secant PQ. Formally, we say the slope of the tangent to the graph y = f(x) at point P(a, f(a)) is $\boxed{m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$, if this limit exists.

In our investigation, we found that the slope of the tangent to a curve at a point P is the limiting slope of the secant PQ as the point Q slides along the curve toward P. In other words, the slope of the tangent is said to be the ***limit*** of the slope of the secant as Q approaches P along the curve.

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Example 1: Determine the slope of the tangent to the graph of the parabola f(x) = x^2 as P (3,9).

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$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(3+h)^2 - (3)^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{9} + 6h + h^2 \cancel{- 9}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (6+h)$$

$$= 6$$

Example 2: a) using the definition of the slope of a tangent, determine the slop of the tangent to the curve y = $-x^2 + 4x + 1$ at the point determined by x = 3. a

$$m = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$f(3+h) = -(3+h)^2 + 4(3+h) + 1$$
$$= -(9+6h+h^2) + 12 + 4h + 1$$
$$= -9 - \cancel{6}h - h^2 + 12 + \cancel{4}h + 1$$
$$= -h^2 - 2h + 4$$

$$f(3) = -(3)^2 + 4(3) + 1$$
$$= -9 + 12 + 1$$
$$= 4$$

$$m = \lim_{h \to 0} \frac{\overbrace{(-h^2 - 2h + 4)}^{f(3+h)} - \overbrace{4}^{f(3)}}{h}$$

$$m = \lim_{h \to 0} \frac{-h^2 - 2h}{h}$$

$$m = \lim_{h \to 0} \frac{-\cancel{h}(h+2)}{\cancel{h}}$$

$$m = \lim_{h \to 0} -h - 2$$

$$m = -2$$

b) Determine the equation of the tangent.

$y = mx + b$

$m = -2$
$x = 3$
$y = 4$

Example 3: Determine the slope of the tangent to the rational function $f(x) = \frac{3x+6}{x}$ at the point (2,6).

$a \quad f(a)$

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3(2+h)+6}{(2+h)} - 6}{h}$$

$$= \lim_{h \to 0} \frac{\frac{6+3h+6}{2+h} - \frac{6(2+h)}{2+h}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{12}+3h-\cancel{12}-6h}{2+h}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-3h}{2+h}}{h}$$

$$= \lim_{h \to 0} \frac{-3h}{2+h} \times \frac{1}{h} \qquad \frac{-3\cancel{h}}{\cancel{h}(2+h)}$$

$$= \lim_{h \to 0} \frac{-3}{2+h}$$

0 works now!

$$= \frac{-3}{2} = -1.5$$

Example 4: Find the slope of the tangent to $f(x) = \sqrt{x}$ at x = 9. a

$$m = \lim_{h \to 0} \frac{\sqrt{9+h} - \sqrt{9}}{h}$$

rationalize numerator

$$m = \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$$

$$m = \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h} \times \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}$$

$$m = \lim_{h \to 0} \frac{\cancel{9} + \cancel{h} \cancel{- 9}}{\cancel{h}(\sqrt{9+h} + 3)}$$

$$m = \lim_{h \to 0} \frac{1}{\sqrt{9+h} + 3}$$

0?

$$= \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{6}$$

Consolidation

Practice

Pg. 18

Make sure you can solve the problems in

#6, 8 - 11

Application problem: **#20**

TIPS problem: **#21**