

Learning Goal: I will be able to determine if the limit of a function exists and, if it does exist, what the value of the limit is. I will be able to determine if a function is continuous.

Minds On: Describe the function

Action: Note and examples

Consolidation: Exit Question

Minds On

What's the rate of change?

The height of an object t seconds after being dropped is $h(t) = -4.905t^2 + h_0$ where h_0 is the initial height of the object in metres.

At what speed is an object travelling 3 seconds after being dropped from a height of a

a. 5 metres?

b. 10 metres?

c. 20 metres?

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\text{roc} = \lim_{h \rightarrow 0} \frac{(-4.905(3+h)^2 + 10) - (-4.905(3)^2 + 10)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(-4.905)(9+6h+h^2) + 10 + 44.145 - 10}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{44.145} - 29.43h - 4.905h^2 + \cancel{10} + \cancel{44.145} - \cancel{10}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-4.905h^2 - 29.43h}{h}$$

$$= \lim_{h \rightarrow 0} -4.905h - 29.43$$

$$= -29.43 \text{ m/s}$$

Minds On

Describe the function and its graph:

$$\frac{x^2 - 1}{x - 1}$$

hole at
 $x=1$

$$y = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

Action

The Limit of a Function

What's a limit?

Action

The Limit of a Function

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$$

Key Ideas

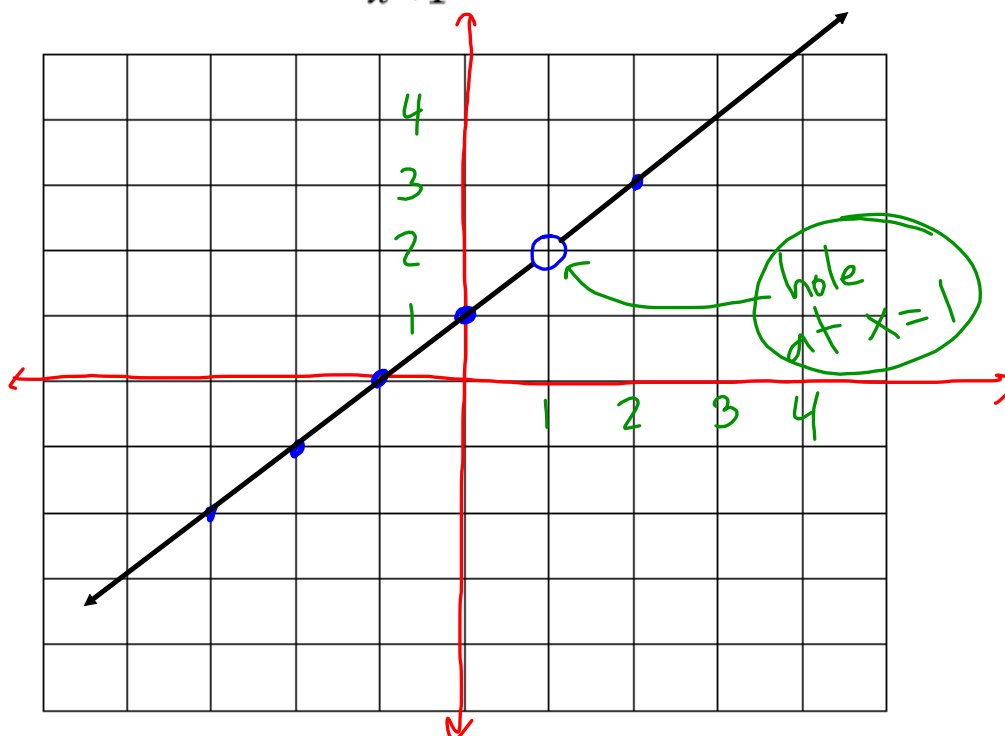
- The limit of a function $y = f(x)$ at $x = a$ is written as: $\lim_{x \rightarrow a} f(x) = L$, which means that $f(x)$ approaches the value L as x approaches the value a from both the left and right side.
- $\lim_{x \rightarrow a} f(x)$ may exist even if $f(a)$ is not defined.
- $\lim_{x \rightarrow a} f(x)$ can be equal to $f(a)$. In this case, the graph of $f(x)$ passes through the point $(a, f(a))$.
- If $\lim_{x \rightarrow a^-} f(x) = L$ and $\lim_{x \rightarrow a^+} f(x) = L$, then L is the limit of $f(x)$ as x approaches a , that is $\lim_{x \rightarrow a} f(x) = L$. Otherwise, $\lim_{x \rightarrow a} f(x)$ does not exist.

from the left

from the right

Action

Example 1: Determine $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ by graphing.



As x approaches 1 from the left, the function approaches 2.

As x approaches 1 from the right, the function approaches 2.

Therefore,

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$$

Action

Example 2: Determine $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ by using a table.

	x approaches 1 from the left \rightarrow						\leftarrow x approaches 1 from the right				
x	0	0.5	0.9	0.99	0.999	1	1.001	1.01	1.1	1.5	2
$\frac{x^2 - 1}{x - 1}$	1	1.5	1.9	1.99	1.999	undefined	2.001	2.01	2.1	2.5	3
$f(x) = \frac{x^2 - 1}{x - 1}$ approaches 2 from below						$f(x) \rightarrow 2$ from above					

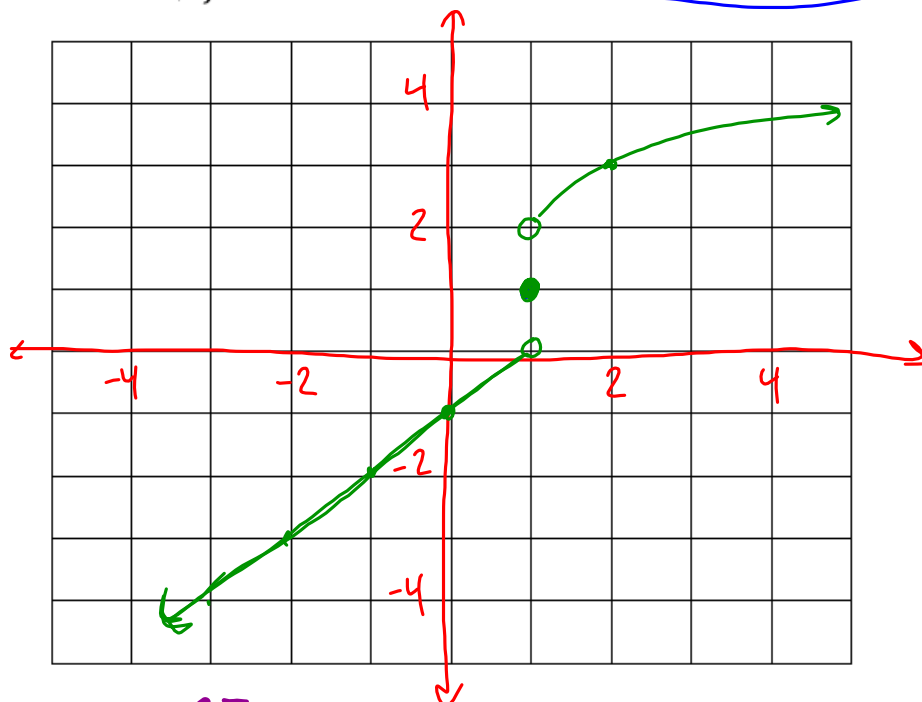
$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$$

Action

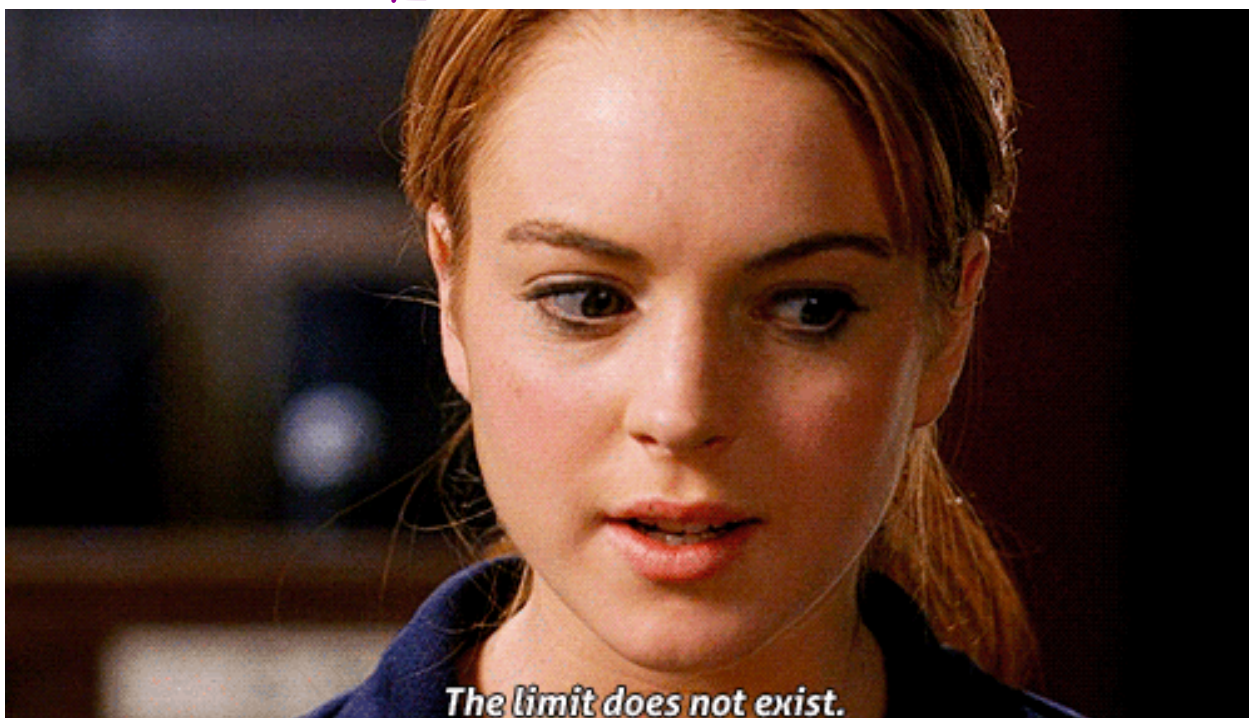
Example 3: Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} x - 1, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2 + \sqrt{x - 1}, & \text{if } x > 1 \end{cases}$$

Determine $\lim_{x \rightarrow 1} f(x)$



As $x \rightarrow 1^-$ $f(x) \rightarrow 0$
 As $x \rightarrow 1^+$ $f(x) \rightarrow 2$



CONSOLIDATION

What's the limit?

CONSOLIDATION

Practice

Pg. 37

4, 6, 7, 11, 12