

3.1 Higher Order Derivatives

The derivative of the derivative function is called the second derivative. If the first derivative is $y = f'(x)$, we write the second derivative as $f''(x)$.

Example 1: Determine the second derivative of $f(x) = \frac{x}{1+x}$ when $x = 1$.

Example 2: An object is moving along a straight line. Its position, $s(t)$, to the right of a fixed point is given by the graph shown. When is the object moving to the right, when is it moving to the left, and when is it at rest?

Example 3: The position of an object moving on a line is given by $s(t) = 6t^2 - t^3$, $t \geq 0$, where s is in metres and t is in seconds.

- a) Determine the velocity and acceleration of the object at $t = 2$.
- b) At what time(s) is the object at rest?
- c) In which direction is the object moving at $t = 5$?
- d) When is the object moving in a positive direction?
- e) When does the object return to its initial position?

Example 4: Discuss the motion of an object moving on a horizontal line if its position is given by $s(t) = t^2 - 10t$, $0 \leq t \leq 12$, where s is in metres and t is in second. Include the initial velocity, final velocity, and any acceleration in your discussion.

Example 5: A baseball is hit vertically upward. The position function $s(t)$, in metres of the ball above the ground is $s(t) = -5t^2 + 30t + 1$, where t is in seconds.

- a) Determine the maximum height reached by the ball.
- b) Determine the velocity of the ball when it is caught 1 m above the ground.

- When we take the first derivative of the position function, $s(t)$, it represents the velocity of the object at time t . $v(t) = s'(t) = \frac{ds}{dt}$
- When we take the second derivative of the position function, $s(t)$, it represents the acceleration of the object at time t . $a(t) = v'(t) = s''(t)$, or $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
- Negative velocity indicates that an object is moving in a negative direction at time t .
- Negative acceleration indicates that the velocity is decreasing at time t .
- An object is accelerating (speeding up) when its velocity and acceleration have the same signs.
- An object is decelerating (slowing down) when its velocity and acceleration have opposite signs.
- The speed of an object is the magnitude of its velocity at time t .