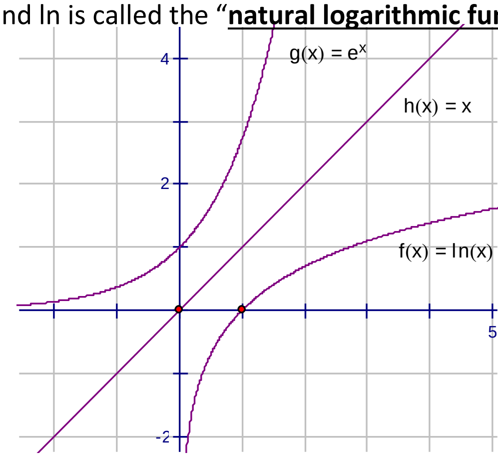
5.1 Derivatives of Exponential Functions

Exponential functions are often used to model rapid change. We are going to look at the function $y = e^x$ and its derivative. The number 'e' is a special irrational number, just like π. It is also referred to as the "**natural number**".

We learned in Advanced Functions that the inverse of the exponential function is the logarithmic function. For example, the inverse of $y = 2^x$ is $y = \log_2 x$. The inverse of $y = e^x$ would be $y = \log_e x$. This can also be written as $y = \ln x$, and ln is called the "**natural logarithmic function**".

Properties of y = e^x



$y = e^x$	$y = \ln x$
The domain is {x ∈ R}	The domain is {x ∈ R, x > 0}.
The range is {y ∈ R, y > 0}	The range is {y ∈ R}
The function passes through (0,1)	The function passes through (1,0)
$e^{\ln x} = x, x > 0$	$\ln e^x = x, x \in R$
The line y = 0 is the horizontal asymptote	The line x = 0 is the vertical asymptote

Derivative of $f(x) = e^x$

For the function $f(x) = e^x$, $f'(x) = e^x$

Derivative of a Composite Function Involving e^x

In general, if $f(x) = e^{g(x)}$, then $f'(x) = e^{g(x)}g'(x)$ by the chain rule

Example 1: Determine the derivative of $f(x) = e^{3x}$.

Example 2: Determine the derivative of each function.

a) $g(x) = e^{x^2 - x}$

b) f(x) = x^2e^x

Example 3: Given $f(x) = 3e^{x^2}$, determine f'(-1).

Example 4: Determine the equation of the line tangent to $y = \frac{e^x}{x^2}$, where x = 2.