Learning Goal: I will be able to interpret different solutions to linear systems.

Minds On: Find the POI

Action: Note: 9.2

Consolidation: Planning

Minds On

Solve the following system of equations for x, y & z.

$x - y + z = 1$ $2x + y - z = 11$ $3x + y + 2z = 12$

Try this as you come in.

Minds On

Solve the following system of equations:

① $2x + y = -9$ ② $x + 2y = -6$

Rearrange ①

③ $y = -9 - 2x$

Sub ③ into ②

$x + 2(-9 - 2x) = -6$

$x - 18 - 4x = -6$

$-3x = 12$

$\boxed{x = -4}$

Sub $x = -4$ into ③

$y = -9 - 2(-4)$

$\boxed{y = -1}$

$\therefore (x, y) = (-4, -1)$

A linear system of equations can have zero (parallel), one (a single POI), or an infinite number of solutions (coincident lines).

Example 1: The number of solutions to the following system of equations depends on the value(s) of a and b. Determine values of a and b for which this system has no solutions, an infinite number of solutions, and one solution.

$x + 4y = a$ $x + by = 8$

No Solutions

$b = 4$

$a \neq 8$

Infinite Solutions

$a = 8$

$b = 4$

One Solution

$b \neq 4$

Example 2: Determine whether x = -3, y = 5, and z = 6 is a solution to the following system:

① $2x + 3y - 5z = -21$ ② $x - 6y + 6z = 8$

① LS = $2x+3y-5z$
$= 2(-3)+3(5)-5(6)$
$= -6+15-30$
$= -21$
RS $= -21$ ✓

$\therefore (-3,5,6)$ is on the first plane

② LS = $x-6y+6z$
$= (-3)-6(5)+6(6)$
$= -3-30+36$
$= 3$
RS $= 8$ ✗

$\therefore (-3,5,6)$ is not on the 2nd plane and is not a solution to the system

Consistent and Inconsistent Systems of Equations

A system of equations is consistent if it has either one solution or an infinite number of solutions. A system is inconsistent if it has no solutions.

Example 3: Solve the following system of equations for x, y & z.

① $x - y + z = 1$ ② $2x + y - z = 11$ ③ $3x + y + 2z = 12$

1. List the equations. Place an equation starting with a coefficient of 1 on top.

① $x-y+z=1$
② $2x+y-z=11$
③ $3x+y+2z=12$

2. Use the top equation to eliminate x from the other equations.

① $x-y+z=1$
④ $0x+3y-3z=9$ $-2\times①+②$
⑤ $0x+4y-z=9$ $-3\times①+③$

3. Get the leading coefficient of the middle equation to 1. Use that to eliminate y from the bottom equation.

① $x - y + z = 1$

⑥ $0x + y - z = 3$ $\frac{1}{3} \times$ ④

⑦ $0x + 0y + 3z = -3$ $-4 \times$ ⑥ $+$ ⑤

* we have created a triangle of zeros in the bottom left corner.

4. Back substitute to solve for x, y, z.

From ⑦ $3z = -3$

$\boxed{z = -1}$

From ⑥

$$y - z = 3$$
$$y - (-1) = 3$$
$$y + 1 = 3$$
$$\boxed{y = 2}$$

From ①

$$x - y + z = 1$$
$$x - (2) + (-1) = 1$$
$$x - 2 - 1 = 1$$
$$\boxed{x = 4}$$

$\therefore (x, y, z) = (4, 2, -1)$

5. Perform checks on the original equations.

Example 4: Solve the following system of equations.

① $x - 3y - 2z = -9$
② $2x - 5y + z = 3$
③ $-3x + 6y + 2z = 8$

① $x - 3y - 2z = -9$
④ $0x + y + 5z = 21$ $-2 \times ① + ②$
⑤ $0x - 3y - 4z = -19$ $3 \times ① + ③$

① $x - 3y - 2z = -9$
④ $0x + y + 5z = 21$
⑥ $0x + 0y + 11z = 44$ $3 \times ④ + ⑤$

From ⑥ $11z = 44$
$z = 4$

From ④ $y + 5(4) = 21$
$y = 1$

From ①

$$x - 3(1) - 2(4) = -9$$

$$x = -9 + 11$$

$$\boxed{x = 2}$$

$$\therefore (x, y, z) = (2, 1, 4)$$

Example 5: Solve the following system of equations.

① $x + 4y - 3z + 6 = 0$

② $2x + 8y - 6z + 11 = 0$

① $x + 4y - 3z + 6 = 0$

③ $0x + 0y + 0z - 1 = 0$ $-2 \times ① + ②$

$-1 = 0$

∴ these planes never meet (they are parallel)