**Learning Goal:** I will be able to determine the shortest distance from a point to a plane.

**Minds On:** What are the formulae?

**Action:** Distance Formulae II

**Consolidation:** Practice Time?

## Minds On

### Yesterday's Equations

Yesterday we were introduced to two equations:

1. The distance from a point to a line in $R^2$.

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

2. The distance from a point to a line in $R^3$.

$$d = \frac{|\vec{m} \times \vec{QP}|}{|\vec{m}|}$$

What do you think the equation for the distance from a point to a plane would be?

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

# 9.6 The Distance from a Point to a Plane

Distance from a Point $P_0(x_0, y_0, z_0)$ to the plane with Equation $Ax + By + Cz + D = 0$

$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$, where d represents the distance between point $P_0(x_0, y_0)$, and the plane defined by $Ax + By + Cz + D = 0$.

***Example 1:*** Determine the distance from point P(-1, 2, 4) to the plane $8x - 4y + 8z + 3 = 0$.

$$d = \frac{|(8)(-1) - 4(2) + 8(4) + 3|}{\sqrt{(8)^2 + (-4)^2 + (8)^2}}$$

$$d = \frac{|-8 - 8 + 32 + 3|}{\sqrt{144}}$$

$$d = \frac{|19|}{12}$$

$$d = \frac{19}{12}$$

$$d \doteq 1.58$$

http://hotmath.com/learning_activities/interactivities/3dsystems.swf

***Example 2:*** a) Determine the distance between the two planes $\pi_1: 2x - y + 2z + 4 = 0$ and $\pi_2: 2x - y + 2z + 16 = 0$.

From $\Pi_1$: Let $x, y = 0$

$$2z + 4 = 0$$

$$\boxed{z = -2}$$

$\therefore$ point on $\Pi_1 = (0, 0, -2)$

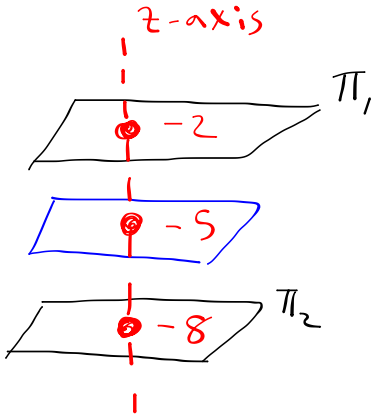
$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$d = \frac{|2(0) - (0) + 2(-2) + 16|}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$$

$$d = \frac{12}{\sqrt{9}}$$

$$d = \frac{12}{3} = 4$$

b) Determine the equation of the plane that is equidistant from $\pi_1$ and $\pi_2$.




From $\pi_1$: z intercept $= -2$

From $\pi_2$: z intercept $= -8$

z-intercept of the plane halfway between is $(0,0,-5)$

Cartesian Equation: $2x - y + 2z + D = 0$

$2(0) - (0) + 2(-5) + D = 0$

$D = 10$

$\therefore$ the equation is $2x - y + 2z + 10 = 0$

***Example 3:*** Determine the distance between $L_1: \vec{r} = (-2,1,0) + s(1,-1,1), s \in R$ and $L_2: \vec{r} = (0,1,0) + t(1,1,2), t \in R$.

1. Create two parallel planes using the lines.

$\Pi_1: \vec{r} = (-2,1,0) + s(1,-1,1) + t(1,1,2)$

$\Pi_2: \vec{r} = (0,1,0) + t(1,1,2) + s(1,-1,1)$

* $\Pi_1 \parallel \Pi_2$

2. Find Cartesian equation of $\Pi_2$

$\vec{n} = \vec{m}_1 \times \vec{m}_2$

$$\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} \quad x = -3$$

$$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \quad y = -1$$

$$\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} \quad z = 2$$

$\vec{n} = (-3,-1,2)$

***Example 3:*** Determine the distance between $L_1: \vec{r} = (-2,1,0) + s(1,-1,1), s \in R$ and $L_2: \vec{r} = (0,1,0) + t(1,1,2), t \in R$.

Let P represent any point on $L_1$

$P = (-2+s, 1-s, s)$

Let Q represent any point on $L_2$

$Q = (t, 1+t, 2t)$

Find $\overrightarrow{PQ} = (t+2-s, t+s, 2t-s)$

When $\overrightarrow{PQ} \perp$ to $L_1$ and $\overrightarrow{PQ} \perp$ to $L_2$ we have the points that minimize the distance between.

$\overrightarrow{PQ} \cdot \vec{m}_1 = 0$

$(t+2-s, t+s, 2t-s) \cdot (1,-1,1) = 0$

$t+2-s-t-s+2t-s = 0$

$2t+2-3s = 0$ (1)

$\overrightarrow{PQ} \cdot \vec{m}_2 = 0$

$(t+2-s, t+s, 2t-s) \cdot (1,1,2) = 0$

$t+2-s+t+s+4t-2s = 0$

$6t+2-2s = 0$ (2)

① $2t + 2 - 3s = 0$

② $6t + 2 - 2s = 0$

① $2t + 2 - 3s = 0$

③ $0t - 4 + 7s = 0$

$-3 \times ① + ②$

From ③ $7s = 4$

$$\boxed{s = \frac{4}{7}}$$

Sub $s = \frac{4}{7}$ into ①

$2t + 2 - 3\left(\frac{4}{7}\right) = 0$

$2t + 2 - \frac{12}{7} = 0$

$$2t = \frac{12}{7} - \frac{14}{7}$$

$$2t = \frac{-2}{7}$$

$$t = \frac{-2}{14}$$

$$\boxed{t = -\frac{1}{7}}$$

From L: $\vec{r} = (-2,1,0) + s(1,-1,1)$

$$x = -2 + \frac{4}{7} = \frac{-10}{7}$$

$$y = 1 - \frac{4}{7} = \frac{3}{7}$$

$$z = \frac{4}{7} = \frac{4}{7}$$

$$\boxed{\left(-\frac{10}{7}, \frac{3}{7}, \frac{4}{7}\right)}$$

From $L_2$: $\vec{r} = (0,1,0) + t(1,1,2)$

$x = -\frac{1}{7}$

$y = 1 - \frac{1}{7} = \frac{6}{7}$

$z = -\frac{2}{7}$

$\left(-\frac{1}{7}, \frac{6}{7}, -\frac{2}{7}\right)$

Distance between the points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$d = \sqrt{\left(-\frac{1}{7} + \frac{10}{7}\right)^2 + \left(\frac{6}{7} - \frac{3}{7}\right)^2 + \left(\frac{-2}{7} - \frac{4}{7}\right)^2}$$

$$d = \sqrt{\left(\frac{9}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2}$$

$$d = \sqrt{\frac{81}{49} + \frac{9}{49} + \frac{36}{49}}$$

$$d = \sqrt{\frac{126}{49}}$$

$$d = \frac{\sqrt{126}}{7}$$

$$d \doteq 1.60$$

$$Ax + By + Cz + D = 0$$

$$(-3)(0) + (-1)(1) + 2(0) + D = 0$$

$$D = 1$$

$$\Pi_2 : -3x - y + 2z + 1 = 0$$

$$\text{Point} : (-2, 1, 0)$$

$$d = \frac{|(-3)(-2) + (-1)(1) + (2)(0) + 1|}{\sqrt{(-3)^2 + (-1)^2 + (2)^2}}$$

$$d = \frac{6}{\sqrt{14}}$$

$$d = 1.60 \text{ units}$$