What's Going On?

Checking In

Minds on Grade 9 Line Problems

Action! Vector and Parametric Equations

Consolidation What's the Slope?

Learning Goal - I will be able to represent lines in vector and parametric form.

New Unit

Equations of Lines and Planes

Unit Test: Monday, May 28

Minds On

1. Determine the equation of the line through the points (-3, 3) and (3, 7).

Minds On

2. Determine the equation of the line perpendicular to the line from number 1 that goes through the point (2, -6).

$y = \frac{2}{3}x + 5$

$y = -\frac{3}{2}x - 3$

Minds On

3. A line has a slope of 1/3 and goes through the point (4, 5). Without determining the equation of the line, list 3 other points on the line.

rise = 1

run = 3

Action

To find out the **vector** and **parametric** equations of a line, we must be given either two distinct points **or** one point and a vector that defines the direction of the line. In either situation, a **direction vector** for the line is necessary.

A **direction vector** is defined to be a nonzero vector $\vec{m}$ = (a, b) parallel (collinear) to the given line. The direction vector $\vec{m}$ = (a, b) is represented by a vector with a tail at the origin and head at the point (a, b). The x and y components of this direction vector are called its **direction numbers**.

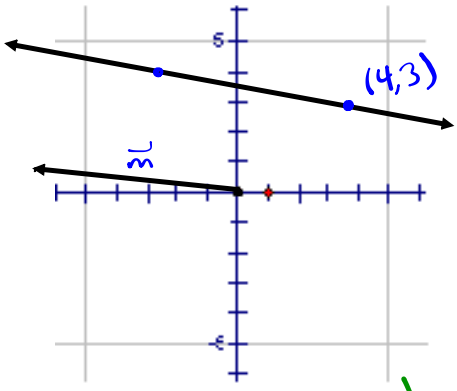
Example 1: a)a line passing through P(4, 3) has $\vec{m}$ = (-7, 1) as its direction vector. Sketch the line.

b) A line passes through the points A(½, -3) and B (¾, ½) . Determine the direction vector for this line, and write it using integer components.

b) $\vec{m} = \overrightarrow{AB}$

$\vec{m} = \left(\frac{3}{4} - \frac{1}{2}, \frac{1}{2} - -3\right)$

$\vec{m} = \left(\frac{1}{4}, \frac{7}{2}\right)$

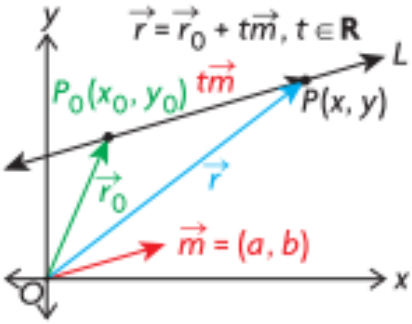


multiply by 4: $\vec{m} = (1, 14)$

The direction for this line is $t(1,14)$ where $t \in \mathbb{R}$

Expressing the Equations of Lines Using Vectors

In general, we would like to determine the equation of a line if we have a direction for the line and a point on it. In the following diagram, the given point $P_0(x_0, y_0)$ is on the line L and is associated with vector $\overrightarrow{OP_0}$, designated as $\vec{r}_0$. The direction of the line is given by $\vec{m} = (a, b)$, where $t\vec{m}$, $t \in \mathbf{R}$ is any vector collinear with $\vec{m}$. $P(x, y)$ represents a general point on the line, where $\overrightarrow{OP}$ is the vector associated with this point.



To find the vector equation of line L, the triangle law of addition is used.

In $\triangle OP_0P$, $\overrightarrow{OP} = \overrightarrow{OP_0} + \overrightarrow{P_0P}$.

Since $\vec{r} = \overrightarrow{OP}$, $\vec{r}_0 = \overrightarrow{OP_0}$, and $t\vec{m} = \overrightarrow{P_0P}$, the vector equation of the line is written as $\vec{r} = \vec{r}_0 + t\vec{m}, t \in \mathbf{R}$.

When writing an equation of a line using vectors, the vector form of the line is sometimes modified and put in parametric form. The parametric equations of a line come directly from its vector equation. How to change the equation of a line from vector to parametric form is shown below.

The general vector equation of a line is $\vec{r} = \vec{r}_0 + t\vec{m}, t \in \mathbf{R}$.

In component form, this is written as $(x, y) = (x_0, y_0) + t(a, b), t \in \mathbf{R}$. Expanding the right side, $(x, y) = (x_0, y_0) + (ta, tb) = (x_0 + ta, y_0 + tb), t \in \mathbf{R}$. If we equate the respective x and y components, the required parametric form is $x = x_0 + ta$

Vector and Parametric Equations of a Line in R^2

Vector Equation: $\vec{r} = \overrightarrow{r_0} + t\vec{m}$, $t \in \mathbf{R}$

Parametric Equations: $x = x_0 + ta$, $y = y_0 + tb$, $t \in \mathbf{R}$

where $\overrightarrow{r_0}$ is the vector from (0, 0) to the point (x_0, y_0) and $\vec{m}$ is a direction vector with components (a, b).

Example 2: a) Determine the vector and parametric equations of a line passing through point A(1, 4) with direction vector $\vec{m} = (-3, 3)$ or $\vec{m} = (-1,1)$

b) Sketch the line, and determine the coordinates of four points on the line. let $t=1$: $(0,5)$ let $t=2$: $(-1,6)$

c) Is either point Q (-21, 23) or point R(-29, 34) on this line?

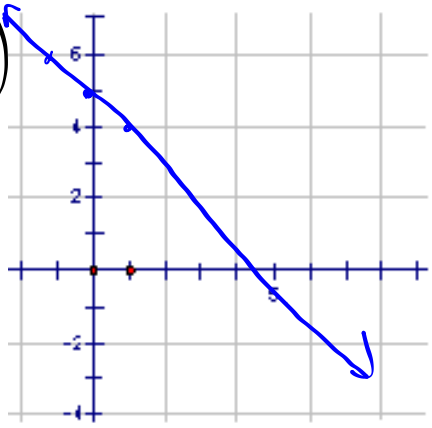
a) $\vec{r} = (1,4) + t(-3,3)$

or

$\vec{r} = (1,4) + t(-1,1)$

$x = 1 - t$ $\}$ $t \in \mathbb{R}$

$y = 4 + t$ $\}$



c) Point Q

$-21 = 1 - t$ $\quad 23 = 4 + t$

$t = 22$ $\quad t = 19$

$\times$ not on the line

Point R

x-value

$-29 = 1 - t$

$t = 30$

y-value

$34 = 4 + t$

$t = 30$

$\therefore (-29, 34)$ is on the line

Example 3: a) Find vector and parametric equations for the line containing points E(-1, 5) and F (6,11).

b) What are the coordinates of the point where this line crosses the x-axis?

c) Can the equation $\vec{r} = (-15, -7) + t(14/3, 4)$, $t \in \mathbf{R}$, also represent the line containing points E and F?

a) $\vec{m} = \overrightarrow{EF} = (6 - -1, 11 - 5)$

$\vec{m} = (7, 6)$

vector

$\vec{r} = (-1, 5) + t(7, 6)$ $t \in \mathbb{R}$

or

$\vec{r} = (6, 11) + t(7, 6)$ $t \in \mathbb{R}$

parametric

$$\left.\begin{aligned} x &= 6 + 7t \\ y &= 11 + 6t \end{aligned}\right\} t \in \mathbb{R}$$

b) on the x-axis, $y = 0$

using parametric equation

$$0 = 11 + 6t$$

$$t = \frac{-11}{6}$$

$$x = 6 + 7\left(\frac{-11}{6}\right)$$

$$x = 6 - \frac{77}{6}$$

$$x = \frac{-41}{6}$$

∴ point is $\left(\frac{-41}{6}, 0\right)$

$\vec{r} = (-15, -7) + s\left(\frac{14}{3}, 4\right)$?

$\vec{r} = (6, 11) + t(7, 6)$

1. Check direction vector $\left(\frac{14}{3}, 4\right)$

$\frac{14}{3}k = 7$

$k = \frac{7(3)}{14}$

$k = \frac{21}{14}$

$k = \frac{3}{2}$

Check the y-coordinates of direction vectors.

$4\left(\frac{3}{2}\right) = \frac{12}{2}$

$= 6$ ✓

∴ direction vectors are equivalent

To check $(-15, -7)$, use parametric equations.

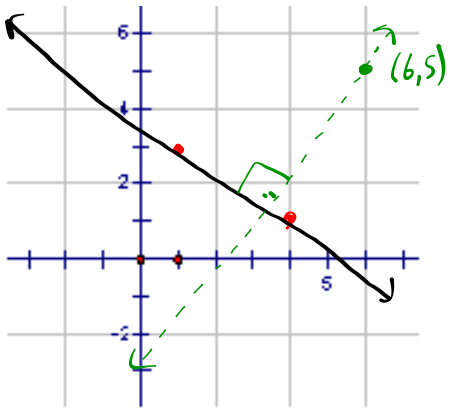
$x = 6 + 7t$ $\quad$ $y = 11 + 6t$

$-15 = 6 + 7t$ $\quad$ $-7 = 11 + 6t$

$t = -3$ ✓ $\quad$ $t = -3$

$\therefore$ the equations are equivalent

Example 4: Determine a vector equation for the line that is perpendicular to $\vec{r} = (4, 1) + s(-3, 2)$, $x \in \mathbf{R}$, and passes through point P(6, 5).



$\vec{r} = (6,5) + t(2,3), t \in \mathbb{R}$

find direction vector

$m_1 = (a,b)$

$(-3,2)\cdot(a,b) = 0$

$-3a + 2b = 0$

$-3a = -2b$

$3a = 2b$

$a = \frac{2b}{3}$

$\therefore m_1 = (2,3)$

$a+b=3,\ a=2$

Consolidation

What's the Slope?

A line has **x = 3 + 3t** and **y = 5 - 2t** as its parametric equations where t ∈ R. What is the slope of the line?

$\vec{r} = (3,5) + t(\overset{x}{3}, \overset{y}{-2})$

$\frac{-2}{3}$

Consolidation

What's the Slope?

A line has $\mathbf{x = x_o + at}$ and $\mathbf{y = y_o + bt}$ as its parametric equations where $t \in R$. What is the slope of the line?