**Learning Goal:** I will be able to work with vector, parametric and symmetric equations of lines in $R^3$.

**Minds On:** Direction Vectors in $R^3$

**Action:** Equations of lines in $R^3$

**Consolidation:** Practice

Minds On

# Recap Question

A line passes through the points A (-2, 6) and B(-4, 2).

a) Determine the vector equation.

$\vec{m} = \overrightarrow{AB} = (-2,-4) = (2,4) = (1,2)$

$\vec{r} = (-2,6) + t(1,2),\ t \in \mathbb{R}$

b) Determine the parametric equations.

$x = -2 + t$
$y = 6 + 2t$ $\Big\}\ t \in \mathbb{R}$

c) Determine the Cartesian equation.

$\therefore \vec{m} = (1,2)$

$\vec{n} = (\overset{A}{2}, \overset{B}{-1})$

$Ax + By + C = 0$

$(2)(-2) + (-1)(6) + C = 0$

$-10 + C = 0$

$C = 10$

$\therefore 2x - y + 10 = 0$

## Minds On

# Direction Vectors in $R^3$

A line passes through the points A (-1, 3, 5) and B(-3, 3, -4). Calculate possible direction vectors for this line.

$\vec{m} = \vec{AB} = (-3 - -1, 3 - 3, -4 - 5)$

$= (-2, 0, -9)$

or $\vec{m} = (2, 0, 9)$

∴ a direction vector for the line is $t(2, 0, 9)$, $t \in \mathbb{R}$

# Action

***Vector and Parametric Equations of Lines in $R^3$***

Vector Equation: $\vec{r} = \overrightarrow{r_o} + t\vec{m}, t \epsilon R$

Parametric Equations: $x = x_0 + ta$, $y = y_0 + tb$, $z = z_0 + tc$, $t \in R$, where $\overrightarrow{r_o} = (x_0, y_0, z_0)$, the vector from the origin to a point on the line and $\vec{m} = (a, b, c)$ is a direction vector of the line.

***Example 2***: Determine the vector and parametric equations of the line passing through P(-2, 3, 5) and Q(-2, 4, -1).

$\vec{m} = \vec{PQ} = (-2 - -2, 4 - 3, -1 - 5)$

$= (0, 1, -6)$

$\vec{r} = (-2, 3, 5) + t(0, 1, -6), \quad t \in \mathbb{R}$

$$\left.\begin{aligned} x &= -2 \\ y &= 3 + t \\ z &= 5 - 6t \end{aligned}\right\} \quad t \in \mathbb{R}$$

***Example 3***: a) Show that the following are vector equations for the same line: $L_1$: $\vec{r}$ = (-1, 0, 4) + s(-1, 2, 5), s ∈ R, and $L_2$: $\vec{r}$ = (4, -10, -21) + m(-2, 4, 10), m ∈ R

a) Show that $L_1 \parallel L_2$

$\vec{m}_2 = 2\vec{m}_1 \therefore L_1 \parallel L_2$

Show that they share a point

From $L_1$: $x = -1 - s$

$y = 2s$

$z = 4 + 5s$

From $L_2$: $(x, y, z) = (4, -10, -21)$

Check $(4,-10,-21)$ in parametric equations for $L_1$.

$(4) = -1 - s$

$s = -5$

$(-10) = 2s$

$s = -5$

$(-21) = 4 + 5s$

$s = -5$ ✓

$\therefore L_1 = L_2$

b) Show that the following are vector equations for different lines: $L_3$: $\vec{r}$ = (1, 6, 1) + p(-1, 1, 2), p ∈ R, and $L_4$: $\vec{r}$ = (-3, 10, 12) + k(1/2, -(1/2), -1), k ∈ R

are they parallel?

$\vec{m}_3 = (-1, 1, 2)$

$\vec{m}_4 = \left(\frac{1}{2}, -\frac{1}{2}, -1\right)$

$\vec{m}_3 = -2\vec{m}_4$ $\therefore L_3 \parallel L_4$

Using $L_3$: $x = 1 - p$

$y = 6 + p$

$z = 1 + 2p$

Sub $(-3, 10, 12)$ into parametric equations of $L_3$

$$(-3) = 1 - p$$
$$p = 4$$

$$(10) = 6 + p$$
$$p = 4$$

$$(12) = 1 + 2p$$
$$p = \frac{11}{2} \quad \times$$

$\therefore L_3 \neq L_4$

## *Symmetric Equations of a Line in $R^3$*

The symmetric equation of a line is derived from using its parametric equations and solving for the parameter in each component, as follows:

vector form: $\vec{r} = \vec{r}_0 + t\vec{m},\ t \in \mathbb{R}$

$\vec{r}_0 = (x_0, y_0, z_0)$

$\vec{m} = (a, b, c)$

parametric form: $x = x_0 + ta \rightarrow t = \dfrac{x - x_0}{a}$

$y = y_0 + tb \rightarrow t = \dfrac{y - y_0}{b}$

$z = z_0 + tc \rightarrow t = \dfrac{z - z_0}{c}$

$$\therefore \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

***Example 4***: Write the symmetric equations of the line passing through each pair of points:

a) A(-1, 5, 7) and B(3, -4, 8)  b) P(-2, 3, 1) and Q(4, 3, -5)

c) X(-1, 2, 5) and Y(-1, 3, 9)

a) $\vec{m} = (4, -9, 1)$

$$\frac{x+1}{4} = \frac{y-5}{-9} = z-7$$

b) $\vec{m} = (6, 0, -6)$ or $\vec{m} = (1, 0, -1)$

$$x+2 = \frac{y-3}{0} = \frac{z-1}{-1}$$

$$x+2 = \frac{z-1}{-1}, \quad y = 3$$

c) $\vec{m} = (0, 1, 4)$

$$y-2 = \frac{z-5}{4}, \quad x = -1$$

## Consolidation

# Solve for k

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9. Determine the value of $k$ for which the direction vectors of the lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z+1}{k-1}$ and $\frac{x+3}{-2} = \frac{z}{1}, y = -1$ are perpendicular.

$\vec{m}_1 = (k, 2, k-1)$

$\vec{m}_2 = (-2, 0, 1)$

if $L_1 \perp L_2$, $\vec{m}_1 \cdot \vec{m}_2 = 0$

$\vec{m}_1 \cdot \vec{m}_2 = (k, 2, k-1) \cdot (-2, 0, 1)$

$0 = -2k + k - 1$

$-k - 1 = 0$

$k = -1$