Learning Goal: I will be able to determine if the limit of a function exits and, if it does exist, what the value of the limit is.
I will be able to determine if a function is continuous.

Minds On: Describe the function

Action: Note and examples

Consolidation: Exit Question

RAFT

Please read until 10:30

Minds On

What's the rate of change?

The height of an object t seconds after being dropped is $\mathbf{h(t) = -4.905t^2 + h_o}$ where h_o is the initial height of the object in metres.

At what speed is an object travelling 3 seconds after being dropped from a height of

a. 5 metres?

b. 10 metres?

c. 20 metres?

$$\text{speed} = \lim_{h \to 0} \frac{h(3+h) - h(3)}{h}$$

$$= \lim_{h \to 0} \frac{\left(-4.905(3+h)^2 + 10\right) - \left(-4.905(3)^2 + 10\right)}{h}$$

$$= \lim_{h \to 0} \frac{\left(-4.905(9+6h+h^2) + 10\right) - \left(-44.145 + 10\right)}{h}$$

$$= \lim_{h \to 0} \frac{-44.145 - 29.43h - 4.905h^2 + 10 + 34.145}{h}$$

$$= \lim_{h \to 0} \frac{-4.905h^2 - 29.43h}{h}$$

$$= \lim_{h \to 0} -4.905h - 29.43$$

$$= -29.43 \text{ m/s}$$

Minds On

Describe the function and its graph:

$$\frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{x-1}$$

hole at $x=1$

Action

The Limit of a Function

the limit as x approaches a of f(x) is L

Key Ideas

- The limit of a function $y = f(x)$ at $x = a$ is written as: $\lim_{x \to a} f(x) = L$, which means that $f(x)$ approaches the value L as x approaches the value a from both the left and right side.

- $\lim_{x \to a} f(x)$ may exist even if $f(a)$ is not defined.

- $\lim_{x \to a} f(x)$ can be equal to $f(a)$. In this case, the graph of $f(x)$ passes through the point $(a, f(a))$.

- If $\lim_{x \to a-} f(x) = L$ and $\lim_{x \to a+} f(x) = L$, then L is the limit of $f(x)$ as x approaches a, that is $\lim_{x \to a} f(x) = L$. Otherwise, $\lim_{x \to a} f(x)$ does not exist.

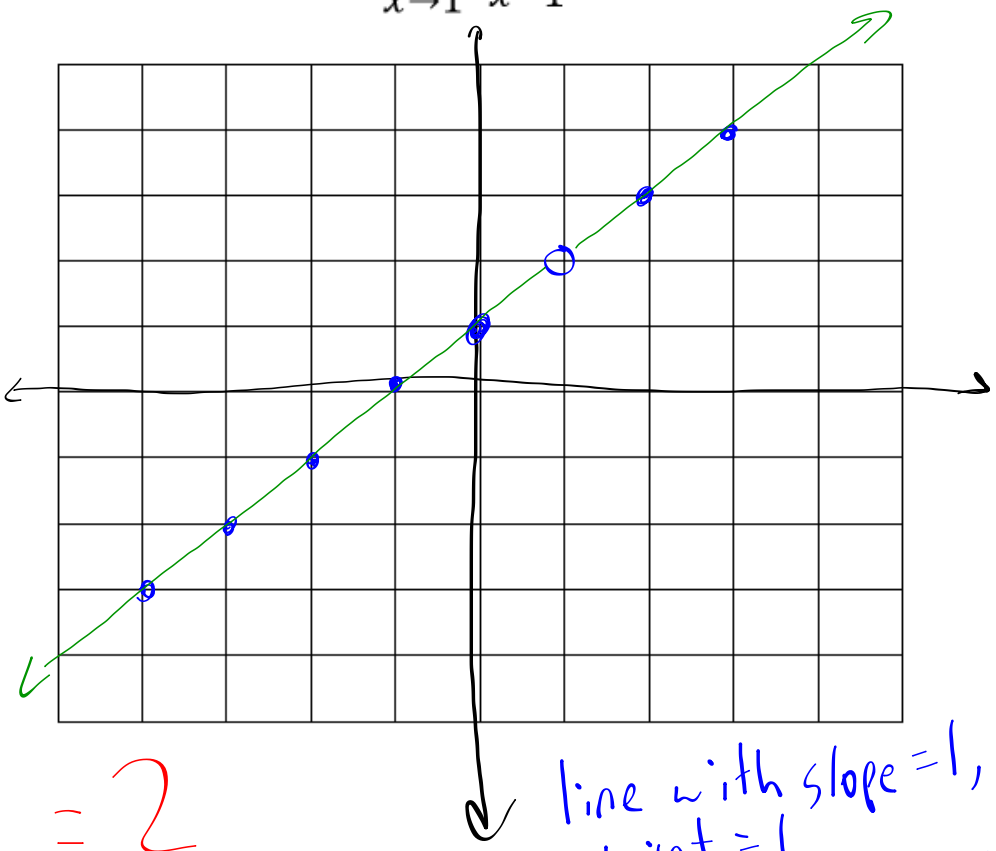
from the left

from the right

Action

$= \dfrac{(x+1)(x-1)}{x-1}$ hole @ $x=1$

Example 1: Determine $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1}$ by graphing.



$\lim\limits_{x\to 1^-} = 2$

$\lim\limits_{x\to 1^+} = 2$

line with slope = 1, y-int = 1 and a hole when $x=1$

$\therefore \lim\limits_{x\to 1} f(x) = 2$

Action

Example 2: Determine $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ by using a table.

x approaches 1 from the left →							←x approaches 1 from the right				
x	0	0.5	0.75	0.9	0.99	1	1.01	1.1	1.25	1.5	2
$\frac{x^2 - 1}{x - 1}$	1	1.5	1.75	1.9	1.99	undefined	2.01	2.1	2.25	2.5	3
f(x) approaches 2 from below							f(x) approaches 2 from above				

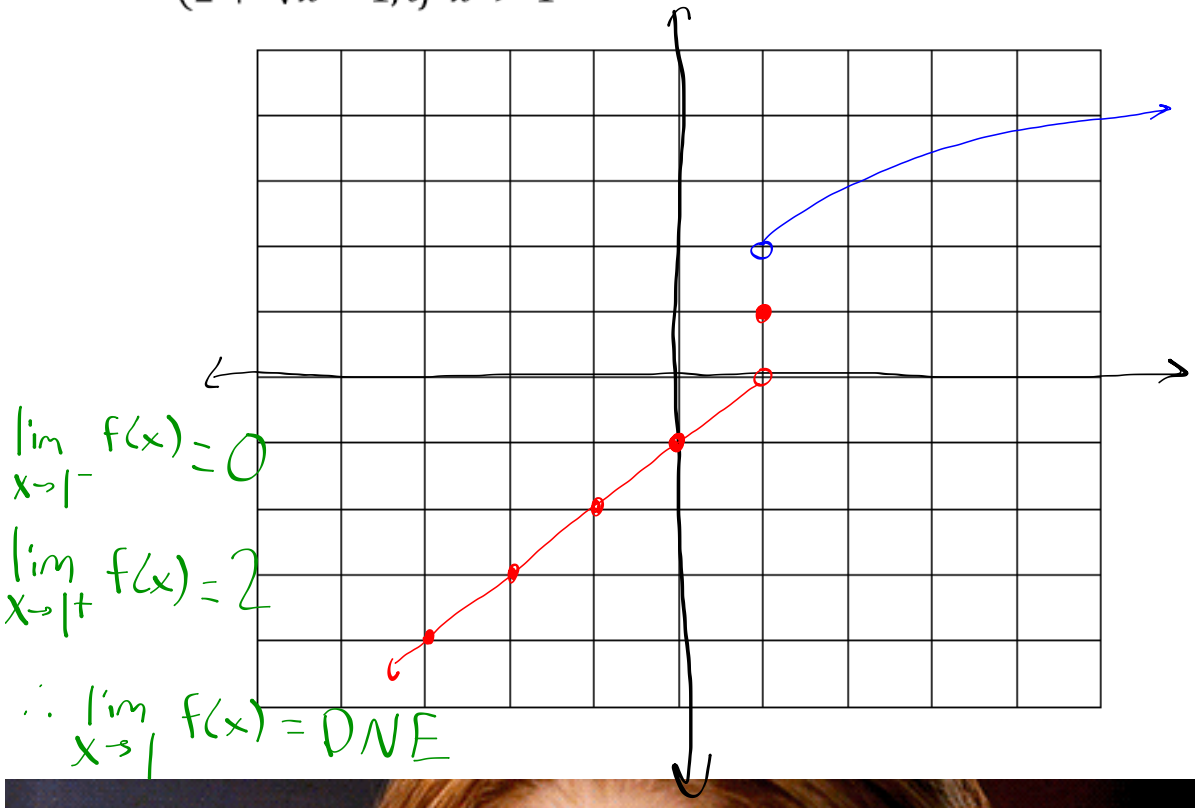
$\therefore \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$

Action

Example 3: Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} x - 1, if\ x < 1 \\ 1, if\ x = 1 \\ 2 + \sqrt{x - 1}, if\ x > 1 \end{cases}$$

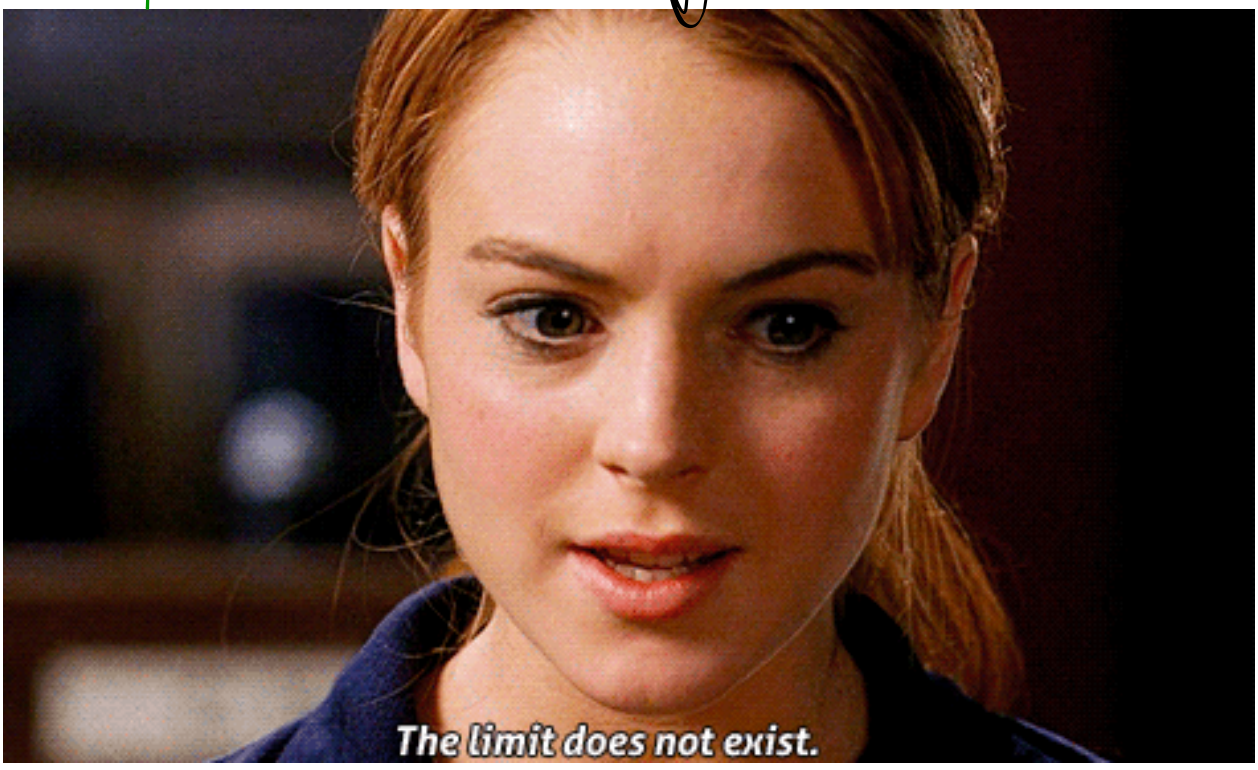
Determine $\lim_{x \to 1} f(x)$



$\lim_{x \to 1^-} f(x) = 0$

$\lim_{x \to 1^+} f(x) = 2$

$\therefore \lim_{x \to 1} f(x) = DNE$



CONSOLIDATION

What's the limit?

$$\lim_{x \to 3} \; 4x - 1 = 11$$

$$\lim_{x \to 2} \; \frac{x+1}{x-2} = \text{DNE}$$

CONSOLIDATION

Practice

Pg. 37

4, 6, 7, 11, 12