Learning Goal: I will be able to identify critical numbers and determine whether they are a max, a a min or neither.

Minds On: Whiteboards - critical points

Action: Class note + practice

Consolidation: Exit Question

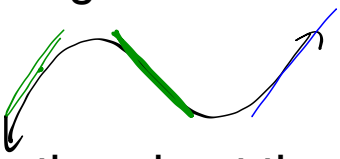
A function f(x) is increasing if: $f'(x) > 0$

A function f(x) is decreasing if: $f'(x) < 0$

Minds On

Draw a function of your choice that has at least one local maximum and one local minimum.

Draw a tangent to the curve on the left of your local max. Draw another tangent to the curve on the right of your local max.



What do you notice about the slopes of these tangents?

slopes change from ⊕ to ⊖

Does this hold true for all local maximums?

Repeat the process for your local min.

slopes change from ⊖ to ⊕

Action

4.2 Critical Points, Local Maxima, and Local Minima

A critical number is a number, c, in the domain of f(x) such that f'(x) = 0 or f'(x) is undefined. As a result, (c, f(c)) is called a critical point and usually corresponds to local or absolute extrema.

The First Derivative Test

Let c be a critical number of a function f. When moving through x-values from left to right:

- If f'(x) changes from negative to positive at c, then (c, f(c)) is a local minimum of f.
- If f'(x) changes from positive to negative at c, then (c, ,f(c)) is a local maximum of f.
- If f'(x) does not change its sign at c, then (c, f(c)) is neither a local min nor a local max.

Remember from Chapter #2 that derivatives that don't exist at cusps and corners on a function's graph. These are also considered extrema, and f'(c) ~~=0~~ is undefined at these points.

Action

when $f'(x) = 0$ or undefined

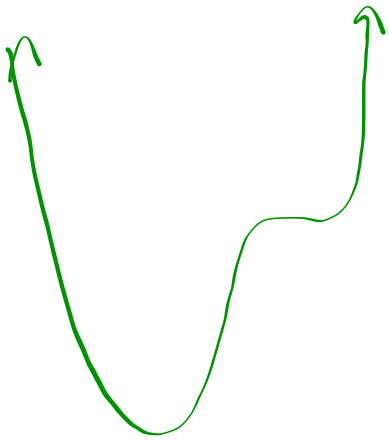
Example 1: For the function $y = x^4 - 8x^3 + 18x^2$, determine all the critical numbers. Determine whether each of these values of x gives a local max, a local min, or neither for the function.

$f'(x) = 4x^3 - 24x^2 + 36x$

$f'(x) = 4x(x^2 - 6x + 9)$

$f'(x) = 4x(x-3)^2$

$\therefore$ critical numbers are $x = 0, 3$



	$x < 0$	$0 < x < 3$	$x > 3$
$4x$	$-$	$+$	$+$
$(x-3)^2$	$+$	$+$	$+$
y'	$-$	$+$	$+$
shape	\	/	/

$\therefore$ local min. when $x = 0$, and neither a min. nor a max. when $x = 3$.

Action

Example 2: Determine whether or not the function $f(x) = x^3$ has a max or min at $(c, f(c))$, where $f'(c) = 0$.

$f'(x) = 3x^2$

$0 = 3x^2$

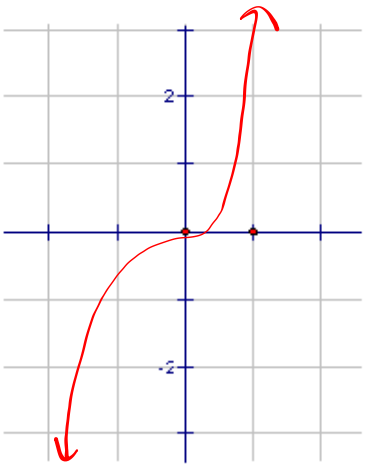
$x = 0$

critical is $x = 0$

$f'(-1) = +$

$f'(1) = +$

neither a max nor a min



Action

Example 3: For the function $f(x) = (x + 2)^{2/3}$, determine the critical numbers. Use your calculator to sketch a graph of the function.

$f'(x) = \frac{2}{3}(x+2)^{-\frac{1}{3}}$

$f'(x) = \frac{2}{3\sqrt[3]{x+2}}$

$0 = \frac{2}{3\sqrt[3]{x+2}}$ → $0 = 2$ $f'(x)$ never $= 0$

$f'(x)$ is undefined when $x = -2$

* but $f(-2) = 0$ and it exists

	$x < -2$	$x > -2$	
$3\sqrt[3]{x+2}$	−	+	
y'	−	+	

*because f'(x) is undefined when x = -2, we have a critical number x = -2.

When x = -2, f(x) = 0.

Therefore our critical point is (-2,0)

To determine if we have a cusp or a corner, we will test the behaviour of the function as we approach $x = -2$ from either side.

From the left:
Test $f'(x)$ when $x = -2.1, -2.01, -2.001$

From the right
Test $f'(x)$ when $x = -1.9, -1.99, -1.999$

$f'(-2.1) = -1.44$

$f'(-2.01) = -3.09$

$f'(-2.001) = -6.67$

$f'(-1.9) = 1.44$

$f'(-1.99) = 3.09$

$f'(-1.999) = 6.67$

As we approach $x = -2$ from the left, the function gets steeper in the negative direction.

As we approach $x = -2$ from the right, the function gets steeper in the positive direction.

$\therefore$ at $x = -2$ we have a cusp!

Action

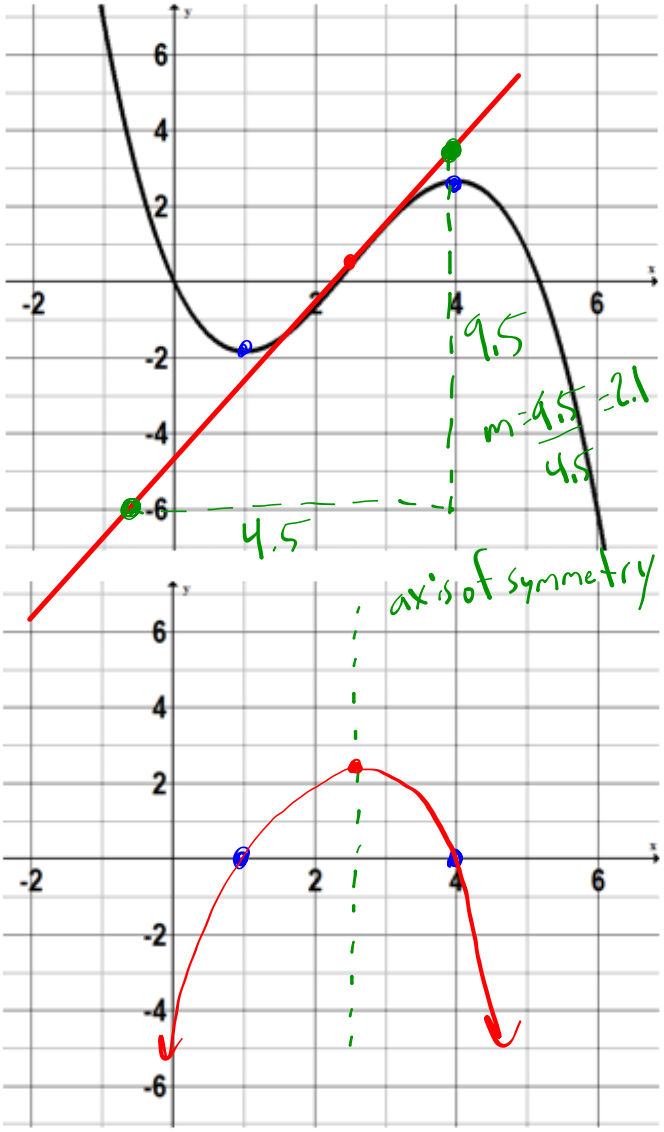
Example 4: Given the graph of a polynomial function y = f(x), graph y = f'(x).

Where f(x) has turning points, f'(x) has zeroes (because slope of f(x) = 0)

When f(x) is ↓, f'(x) is ⊖

When f(x) is ↑, f'(x) is ⊕

To find y-value of vertex for derivative function, find slope of original function when x = 2.5.



Consolidation

Determine the nature of all critical points of f(x) given below, then sketch a rough graph of the function.

$$f(x) = -3x^4 + 8x^3$$

f'(x) exists for all values of x

to find critical numbers, set f'(x) = 0 and solve for x