Learning Goal: I will be able to determine the relationship between the general exponential function and its derivative. I will be able to determine derivatives involving exponentials.

Minds On: Derivative Investigation

Action: Class note + practice

Consolidation: Connect to yesterday

Minds On

Definition of the Derivative

Together we will determine the derivative of

$$f(x) = 2^x$$

using the definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

when $f(x) = 2^x$

$$f'(x) = \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h}$$

$$= \lim_{h \to 0} \frac{2^x(2^h - 1)}{h}$$

$$= 2^x \cdot \lim_{h \to 0} \frac{2^h - 1}{h}$$

let $h = 0.001$

$$= 2^x \cdot 0.69$$

Minds On

Your Turn

We are going to work together to determine an expression for the derivative of

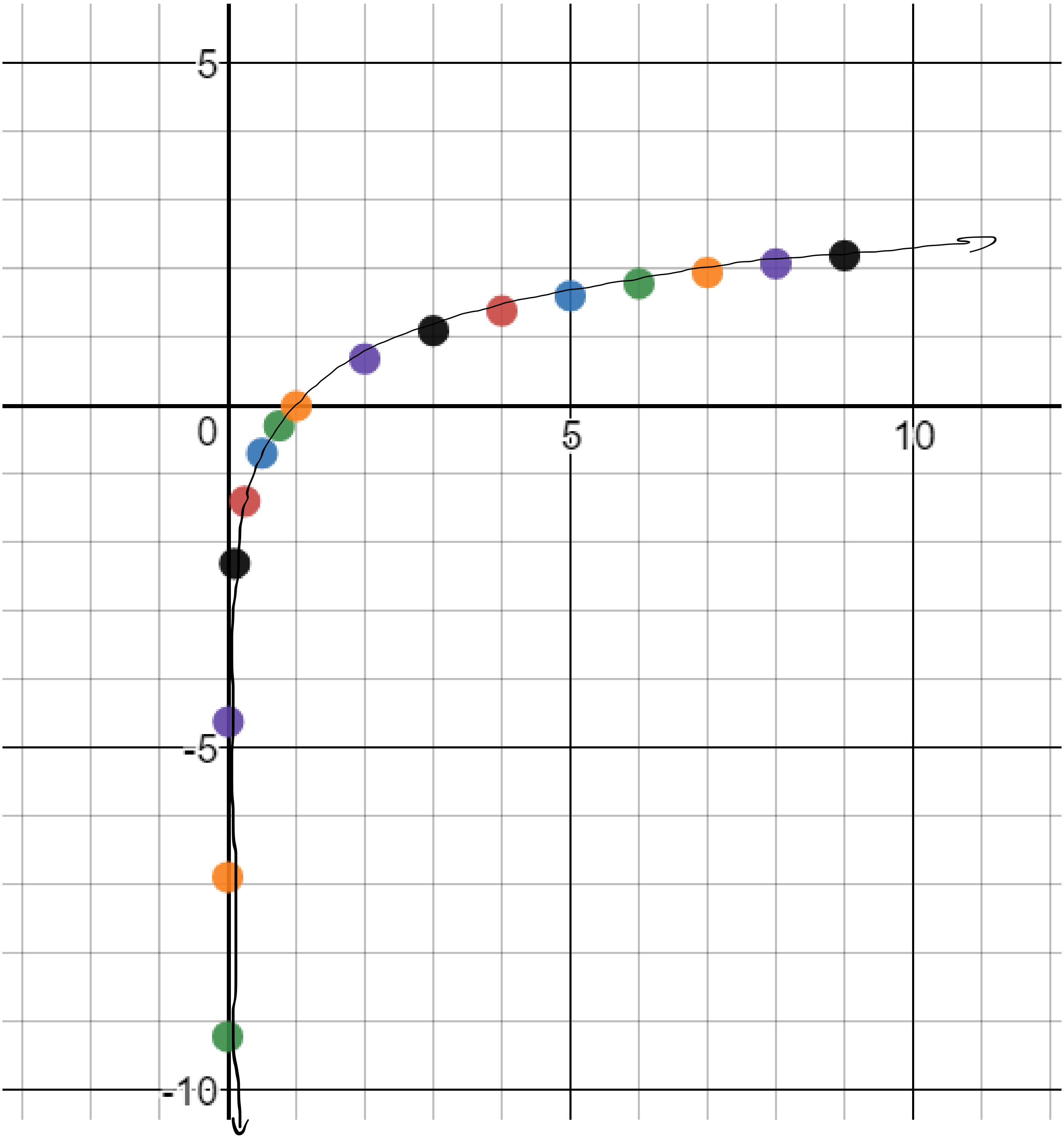
$$\mathbf{f(x) = b^x}$$

We know that the derivative of $\mathbf{f(x) = 2^x}$ is 0.69 b^x. We will refer to the constant 0.69 as *k.*

Determine the value of k for your assigned values of b and fill in the table below.

b	0.0001	0.001	0.01	0.1	0.25	0.5	0.75	1
k	-9.21	-6.96	-4.61	-2.30	-1.39	-0.69	-0.29	0

b	2	3	4	5	6	7	8	9
k	0.69	1.10	1.39	1.61	1.79	1.95	2.06	2.20



Looks like ln x

Action

5.2 The Derivative of the General Exponential Function

Key Ideas

- If $f(x) = b^x$, then $f'(x) = b^x \times \ln b$
- If $f(x) = b^{g(x)}$, then $f'(x) = b^{g(x)} \times \ln b \times g'(x)$
- $\lim_{h \to 0} \frac{b^h - 1}{h} = \ln b$
- When you are differentiating a function that involves an exponential function, use the rules given above, along with the sum, difference, product, quotient, and chain rules as required.

Action

Example 1: Determine the derivatives of

a) $f(x) = 5^x$

$f'(x) = 5^x \cdot \ln 5$

$= \ln 5(5^x)$

b) $f(x) = 5^{3x-2}$

$f'(x) = 5^{3x-2} \times \ln 5 \times 3$

$= 3(5^{3x-2})\ln 5$

c) $3^x x^6$

$f'(x) = 3^x \times \ln 3 \times x^6 + 3^x \cdot 6x^5$

$= 3^x x^5 (x \ln 3 + 6)$

Action

Example 2: On January 1, 1850, the population of Weaverville was 50 000. The size of the population since then can be modelled by the function $P(t) = 50\,000(0.98)^t$, where t is the number of years since January 1st, 1850.

a) What was the population of Weaverville on January 1, 1900?

$$P(50) = 50\,000(0.98)^{50}$$
$$\doteq 18,208$$

b) At what rate was the population of Weaverville changing on January 1, 1900? Was it increasing or decreasing at that time?

$$P'(t) = 50\,000 \times 0.98^t \times \ln 0.98$$
$$P'(50) = 50\,000 \times 0.98^{50} \times \ln 0.98$$
$$\doteq -368$$

$\therefore$ the population was decreasing by 368 ppl/year.

Consolidation

Based on today's lesson about derivatives of exponential functions, determine the derivative of $f(x) = e^{x}$.

$$f'(x) = e^{x} \cdot \ln e$$
$$= e^{x}(1)$$
$$= e^{x}$$