QUADRATICS 1 STUDY PACKAGE

Functions

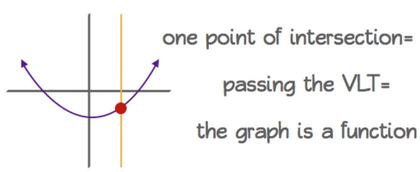
Olive the functions that exist are relations, but olive the rations that exist are not functions. That's punny!

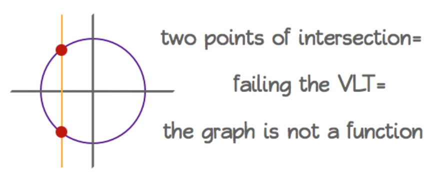
What makes certain relations a function?

- If the relation passes the vertical line test
- If for very value of x, there is only one value of y (set notation)
- If the parabola is vertical

The Vertical Line Test

- If any vertical line passes through more than one point of the graph of a relation, then the relation is not a function
- Examples:





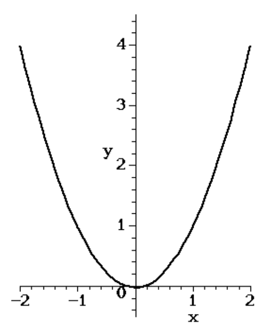
Domain and Range

- Domain: set of the first elements (often the x-values)
- Range: set of the second elements (often the y-values)
- Examples:

{(2, 3), (4, 5), (6, 7), (8, 9)}

Domain: 2, 4, 6, 8

Range: 3, 5, 7, 9



Domain: The set of real numbers

Range: $y \geq 0$

Vertex Form Equations (a, h and k)

Vertex Form: y = a (x – h) ^2 + k

For example, y = 2 (x + 6) ^2 + 4

This means that the a-value of the parabola is 2.

This means that the x-value of the vertex is going to be -6. (If the interval for h is negative, the x-value is positive; if the interval is positive, the x-value is negative)

This means that the y-value of the vertex is going to be 4.

This equation form can be turned into Standard Form by expanding, FOILing and simplifying the equation.

Standard Form: y = ax^2 + bx + c

For example, y = 3x^2 - 5x + 12

This means that the y-intercept is going to be 12.

This equation form can be turned into both vertex and factored form. For vertex form you complete the square and for factored form you factor by grouping.

Factored Form: y = (x – x1) (x – x2)

For example, y = (x – 4) (x – 12)

This means that the “roots”, “zeroes” or “x-intercepts” of the equation are -4 and -12.

In other words, the two points where the parabola touches the x-axis are (4, 0) and (12, 0). There is always the possibility of a parabola not having any roots (where the peak of the parabola is above the x-axis and opens up away from the x-axis), or a parabola having two roots in the same place (in which case the vertex is on the x-axis).

This equation can be turned into standard form by FOILing the brackets.

Completing the Square

Completing the square allows us to get from Standard Form to Vertex Form. Here are the four steps to complete the square when a = 1:

1. Determine what must be added to x^2+bx to make it a perfect square trinomial.

(Square half the coefficient of x)

2. Add and subtract the number found in step 1 to the original equation.

3. Group the perfect square trinomial.

4. Factor the perfect square trinomial and include the remaining constant at the end.

Let's try with a simple example: $y = x^2+4x+6$

1. First we have to find what must be added to x^2+4x to make it a perfect square trinomial. (As stated before, square half of the coefficient of x. In this case x is 4, so we will take 2, and square it. So our number is 4.)

2. Now we add and subtract the number we found in step one, to get the equation:

$y = x^2+4x+4-4+6$

3. Now we find and group the perfect square trinomial. $y = (x^2+4x+4)-4+6$

4. Last, we simplify the equation. We will factor the perfect square trinomial to $(x+2)^2$, which will give us $(x+2)^2-4+6$. Now we finish by simplifying the rest of the equation outside the brackets. Then we will finally have vertex form. Our final vertex form equation is $y = (x+2)^2+2$.

Complete the square to take these equations from Standard Form and get their equivalent in Vertex Form:

a) $y = x^2+10x+13$

c) $y = x^2-8x+25$

b) $y = x^2+6x-19$

d) $y = x^2-12x-4$

What if a is not equal to one though?

1. Factor the coefficient of x^2 from the first two terms.

2. Complete the square as you would when a = 1.

So, let's take our simple example and give x^2 a coefficient, as so: $y = 2x^2+4x+6$

1. First we factor out the new coefficient. Now we will have $y = 2(x^2+2x)+6$

2. Now we have to create a perfect square trinomial, so we take half the coefficient of x, square it, and add it and subtract it from the original equation. We will have $2(x^2+2x+1-1)+6$

3. Next, we have to identify and simplify the perfect square trinomial. So we will change $2[(x^2+2x+1)-1]+6$, and it will become $2[(x+1)^2-1]+6$

4. Now we expand the equation back out. We will have $2(x+1)^2-2+6$.

5. Finally, we simplify the equation. We should end up with $2(x+1)^2+4$ as our final Vertex Form equation.

Now try completing the square on these equations where a is not equal to 1:

a) y = 3x^2+12x+18

c) y = -2x^2+10x-4

b) y = 2x^2+10x-5

d) y = -4x^2-24x-40

First and Second Differences

Good Luck Everyone!

The vertex can be found by finding the co-ordinates in the middle of 2 other co-ordinate sets with the same y-value.

X	Y
-3	-9
-2	-7
-1	-9
0	-15
1	-25
2	-39
3	-57

First Differences	Second Differences
2	-4
-2	-4
-6	-4
-10	-4
-14	-4
-18	-4

If all the second differences are the same, then the parabola is a function!

*Note: When finding 1st and 2nd differences, always subtract the 1st from the 2nd [ie (-57)-(-39)]

Use the second differences and the vertex to create your Vertex Form Equation!

Vertex Form Equation: $-2(x+2)^2-7$

The a value can be found by dividing the 2nd difference in half.

X-value of the vertex

Y-value of the vertex